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General formulation for proton decay rate in minimal supersymmetric SO(10) GUT

T. Fukuyama[1,a], A. Ilakovac[2,b], T. Kikuchi[1,c], S. Meljanac[3,d], N. Okada[4,e]

[1] Department of Physics, Ritsumeikan University, Kusatsu, Shiga, 525-8577 Japan
[2] University of Zagreb, Department of Physics, P.O. Box 331, Bijenička cesta 32, 10002 Zagreb, Croatia
[3] Institut Rudjer Bošković, Bijenička cesta 54, P.O. Box 180, 10002 Zagreb, Croatia
[4] Theory Group, KEK, Oho 1-1, Tsukuba, Ibaraki 305-0801, Japan

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Abstract. We give an explicit formula for the proton decay rate in the minimal renormalizable supersymmetric (SUSY) SO(10) model. In this model, the Higgs fields consist of the **10** and $\overline{\mathbf{126}}$ SO(10) representations in the Yukawa interactions with matter and of the **10**, $\overline{\mathbf{126}}$, **126**, and **210** representations in the Higgs potential. We present all the mass matrices for the Higgs fields contained in this minimal SUSY SO(10) model. Finally, we discuss the threshold effects of these Higgs fields on the gauge coupling unification.

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1 Introduction

Proton decay would be a smoking gun signature for grand unified theories (GUTs). Unfortunately, no such signal has been seen. In fact, very strong experimental limits have been set for this process, placing the minimal GUTs in a very precarious position. SuperKamiokande (SuperK) has set a lower limit on the proton lifetime in the channel $p \to K^+\overline{\nu}$ as follows:

$$\tau(p \to K^+\bar{\nu}) \geq 2.2 \times 10^{33}\,\text{years}, \tag{1}$$

at the 90% confidence level [1]. This has already placed stringent constraints on SU(5). In fact, the minimal renormalizable SUSY SU(5) model is almost absolutely excluded [2].[1] Thus realistic unified model builders must seriously consider the proton lifetime constraints.

Now, SO(10) GUTs have been mainly discussed in connection with the neutrino oscillations since this part reveals the physics beyond the standard model. In this connection, SO(10) GUTs have some advantages over SU(5) GUTs. One of them is that they incorporate the right-handed neutrinos as member of the **16** dimensional spinor representation together with the other standard model fermions and provide a natural explanation of the smallness of the neutrino masses through the seesaw mechanism [4]. In this paper, we consider the minimal renormalizable SUSY SO(10) model. This model contains two Higgs fields, **10** and $\overline{\mathbf{126}}$, in the Yukawa interactions with matter [5,6]. This is a minimal model in the sense that it contains only the renormalizable operators at the GUT scale and it has minimal contents of the Higgs fields compatible with the low-energy experimental data. If we relax the renormalizability at the GUT scale, the different minimal SO(10) models may also possibly be considered [7, 8]. In this paper, we restrict our arguments within the renormalizable theory at the GUT scale and use the word "minimal" in this restricted sense. As was shown in [5, 6], this theory is highly predictive. However, recent data [9,10] showed that one of our predictions, the neutrino mass square ratio, is out of the 3σ allowed region. However if we change very slightly the remaining parameter we can improve the data fitting. Further, incorporating systematically and exhaustively the errors of the input data (quark masses, CKM mixing angles etc.), we can improve the data fitting (to within 3σ as a whole [11]) without changing the model and the seesaw type. Therefore, the minimal model cannot be considered invalid. However, the development of GUTs and rich experimental data drive us to a new stage of precision calculations. That is, we must include not only the uncertainties of the input data but also threshold corrections precisely. In order to investigate the proton decay rate and the gauge coupling unification

[a] e-mail: fukuyama@se.ritsumei.ac.jp
[b] e-mail: ailakov@rosalind.phy.hr
[c] e-mail: rp009979@se.ritsumei.ac.jp
[d] e-mail: meljanac@irb.hr
[e] e-mail: okadan@post.kek.jp

[1] If we take some textures of the mass matrices for fermions and sfermions, we may get a safe region for the proton lifetime in a minimal renormalizable SUSY SU(5) model [3].

in a precise way, we have to determine all the mass spectra of the Higgs fields in terms of the parameters presented in this model. This is a very complicated task itself and is the main motivation of this work. Even in the minimal model, there are many free parameters. So, in a practical analysis of the proton decay rate and also the gauge coupling unification, one has to reduce the number of free parameters. That means that we should consider the smallest number of Higgs contents. Thus, we introduce our Higgs system as the simplest one, $\{\mathbf{10}\oplus\overline{\mathbf{126}}\oplus\mathbf{126}\oplus\mathbf{210}\}$. The meaning of the introduction of these representations will be revealed in the next section. Since our results are the general ones for the "minimal" renormalizable SO(10) models, it can be applicable to any parameter regions. For instance, even if we fix the type of Yukawa couplings in the matter sector and also the Higgs potential, the result is not unique. If we restrict the values of the parameters in the superpotential to some restricted region, we may get the two different types of seesaw mechanisms, type-I [5,6] or type-II [12]. In this paper, we do not explicitly explain the way to save the model from the proton decay rate of SuperK. Our main purpose in this paper is to produce all the mass spectra of the Higgs fields including all the Clebsch–Gordan (CG) coefficients and to propose a general formulation which is applicable to any parameter choices. In these applications, our theory might be found to be insufficient. Even if this is the case, our theory is very useful for a more elaborate theory.

This paper is organized as follows. In Sect. 2, we give the explicit form of the superpotential in our model. In Sect. 3, a very brief description of the symmetry breaking procedure and the decomposition of the original Higgs fields into the minimal supersymmetric standard model (MSSM) are given. In Sect. 4, using these techniques, we can get the mass matrices for a variety of fields, especially for the would-be Nambu–Goldstone (NG) modes. Then we can check that the appropriate NG modes do appear in the mass spectra. In Sect. 5, we check the mass matrices for the electroweak Higgs doublets and consider the conditions for two Higgs doublets to remain light. In Sect. 6, we derive the formulae for the evaluation of the proton decay rate. In Sect. 7, we finally check the remaining mass matrices and the effects of the threshold corrections on the gauge coupling unification. In the appendices, we list all the coefficients of dimension-five and -six operators, which are relevant to proton decay. The applications to a more elaborate model will be given in a separate publication.

2 Minimal SO(10) GUT

In this section, we explain the minimal renormalizable SUSY SO(10) model. As mentioned in the introduction, it contains two Higgs fields in the Yukawa interactions with matter [5,6]. In the SO(10) models, the left- and right-handed fermions in a given ith generation are assigned to a single irreducible representation $\mathbf{16}_i \equiv \Psi_i$. Since $\mathbf{16}\otimes\mathbf{16}=\mathbf{10}\oplus\mathbf{120}\oplus\mathbf{126}$, the fermion masses are generated when the Higgs fields of the $\mathbf{10}$, $\mathbf{120}$, and $\overline{\mathbf{126}}$ dimensional representations develop non-vanishing vacuum expectation values (VEVs). The use of only one Higgs field, $\mathbf{10}$ in the Yukawa interactions with matter, is obviously ruled out for the description of realistic quark and lepton mass matrices. Furthermore, the use of the $\overline{\mathbf{126}}$ dimensional Higgs field has desirable properties for providing masses of the right-handed Majorana neutrinos. Also it was found that $\mathbf{10}\,(\equiv H)$ and $\overline{\mathbf{126}}\,(\equiv\overline{\Delta})$ are suitable for the mass matrices since they satisfy the Georgi–Jarlskog relation. In order to preserve supersymmetry, we must also include the Higgs field Δ of the $\mathbf{126}$ dimensional representation. The Higgs field Φ of the $\mathbf{210}$ dimensional representation is introduced to break the SO(10) gauge symmetry [13] and to make the Higgs doublets included in H and $\overline{\Delta}$ mix [5]. Then the minimal Yukawa coupling becomes

$$W_Y = Y_{10}^{ij}\Psi_i H \Psi_j + Y_{126}^{ij}\Psi_i\overline{\Delta}\Psi_j, \tag{2}$$

and the minimal Higgs superpotential is [13–15]

$$\begin{aligned} W =& m_1\Phi^2 + m_2\Delta\overline{\Delta} + m_3 H^2 \\ &+ \lambda_1\Phi^3 + \lambda_2\Phi\Delta\overline{\Delta} + \lambda_3\Phi\Delta H + \lambda_4\Phi\overline{\Delta}H. \end{aligned} \tag{3}$$

The interactions of $\mathbf{210}$, $\overline{\mathbf{126}}$, $\mathbf{126}$ and $\mathbf{10}$ lead to some complexities in decomposing the GUT representations to the MSSM and in getting the low-energy mass spectra. Particularly, the CG coefficients corresponding to the decompositions of $\mathrm{SO(10)}\to\mathrm{SU(3)}_C\times\mathrm{SU(2)_L}\times\mathrm{U(1)}_Y$ have to be found. This problem was first attacked by Xiao-Gang He and one of the present authors (S.M.) [16] and further by Lee [14]. But they did not present the explicit form of the mass matrices for a variety of Higgs fields and also did not perform a formulation of the proton lifetime analysis. In this paper we will complete that program in the framework of our minimal SO(10) model.

3 Symmetry breaking

In order to discuss the symmetry breaking pattern, here we briefly summarize our conventions for the SO(10) indices. The SO(10) indices $\alpha=1,2,\cdots,9,0$ are divided into two parts, $\alpha=1,2,3,4$ for $\mathrm{SO(4)}\cong\mathrm{SU(2)}\times\mathrm{SU(2)}$ and $\alpha=5,6,7,8,9,0$ for $\mathrm{SO(6)}\cong\mathrm{SU(4)}$. For the $\mathrm{SO(10)}\to\mathrm{SU(3)}_C\times\mathrm{SU(2)_L}\times\mathrm{U(1)}_Y$ decompositions it is very useful to define a "Y diagonal basis": $1'=1+2\mathrm{i}$, $2'=1-2\mathrm{i}$, $3'=3+4\mathrm{i}$, $4'=3-4\mathrm{i}$, $5'=5+6\mathrm{i}$, $6'=5-6\mathrm{i}$, $7'=7+8\mathrm{i}$, $8'=7-8\mathrm{i}$, $9'=9+0\mathrm{i}$, $0'=9-0\mathrm{i}$ (up to a normalization factor, $1/\sqrt{2}$). Hereafter we use this Y diagonal basis and omit the primes: The $\mathbf{10}$ dimensional irreducible representation, H, is spanned by the states $\alpha=1,2,\cdots,9,0$. The $\mathbf{210}$ dimensional irreducible representation, Φ and the $\overline{\mathbf{126}}\oplus\mathbf{126}$ dimensional reducible representation $\overline{\Delta}+\Delta$, are spanned by the antisymmetric tensors of the fourth rank $(\alpha\beta\gamma\delta)$ and the antisymmetric tensors of the fifth rank $(\alpha\beta\gamma\delta\epsilon)$, respectively. Here and below the bracket $(\cdots)$ represents the total anti-symmetrization of the indices within the bracket.

The Higgs fields of the minimal $SO(10)$ model contain five directions which are singlets under $\mathrm{SU(3)}_C\times\mathrm{SU(2)_L}\times$

$\mathrm{U}(1)_Y$. The corresponding VEVs are defined by

$$\langle \Phi \rangle = \sum_{i=1}^{3} \phi_i \, \widehat{\phi}_i, \quad \langle \Delta \rangle = v_{\mathrm{R}} \, \widehat{v_{\mathrm{R}}}, \ \langle \overline{\Delta} \rangle = \overline{v_{\mathrm{R}}} \, \widehat{\overline{v_{\mathrm{R}}}}, \tag{4}$$

where $\widehat{\phi}_i \, (i = 1, 2, 3)$ are included in **210**,

$$\widehat{\phi}_1 = -\frac{1}{\sqrt{24}} (1234) \,, \tag{5}$$

$$\widehat{\phi}_2 = -\frac{1}{\sqrt{72}} (5678 + 5690 + 7890) \,, \tag{6}$$

$$\widehat{\phi}_3 = -\frac{1}{\sqrt{144}} (1256 + 1278 + 1290 \\ + \ +3456 + 3478 + 3490) \,, \tag{7}$$

$\widehat{\overline{v_{\mathrm{R}}}}$ is in $\overline{\mathbf{126}}$,

$$\widehat{\overline{v_{\mathrm{R}}}} = \frac{1}{\sqrt{120}} (13579) \,, \tag{8}$$

and $\widehat{v_{\mathrm{R}}}$ is in **126**,

$$\widehat{v_{\mathrm{R}}} = \frac{1}{\sqrt{120}} (24680) \,. \tag{9}$$

Notice that

$$\widehat{\phi}_i \cdot \widehat{\phi_j} = \delta_{ij} \ \ (i, j = 1, 2, 3) \,, \\ \widehat{\overline{v_{\mathrm{R}}}} \cdot \widehat{\overline{v_{\mathrm{R}}}} = \widehat{v_{\mathrm{R}}} \cdot \widehat{v_{\mathrm{R}}} = 0, \ \widehat{v_{\mathrm{R}}} \cdot \widehat{\overline{v_{\mathrm{R}}}} = 1. \tag{10}$$

Due to the D-flatness condition the absolute values of the VEVs, $\overline{v_{\mathrm{R}}}$ and v_{R} are equal,

$$|\overline{v_{\mathrm{R}}}| = |v_{\mathrm{R}}|. \tag{11}$$

Now we write down the VEV conditions which preserve supersymmetry, with respect to the directions $\widehat{\phi}_1$, $\widehat{\phi}_2$, $\widehat{\phi}_3$, and $\widehat{\overline{v_{\mathrm{R}}}}$, respectively:

$$2 m_1 \phi_1 + 3 \lambda_1 \frac{\phi_3^2}{6\sqrt{6}} + \lambda_2 \frac{v_{\mathrm{R}} \cdot \overline{v_{\mathrm{R}}}}{10\sqrt{6}} = 0, \tag{12}$$

$$2 m_1 \phi_2 + 3 \lambda_1 \left(\frac{\phi_2^2 + \phi_3^2}{9\sqrt{2}} \right) + \lambda_2 \frac{v_{\mathrm{R}} \cdot \overline{v_{\mathrm{R}}}}{10\sqrt{2}} = 0, \tag{13}$$

$$2 m_1 \phi_3 + 3 \lambda_1 \left(\frac{\phi_1 \phi_3}{3\sqrt{6}} + \frac{\sqrt{2}\phi_2 \phi_3}{9} \right) + \lambda_2 \frac{v_{\mathrm{R}} \cdot \overline{v_{\mathrm{R}}}}{10} = 0, \tag{14}$$

$$\left\{ m_2 + \lambda_2 \left(\frac{\phi_1}{10\sqrt{6}} + \frac{\phi_2}{10\sqrt{2}} + \frac{\phi_3}{10} \right) \right\} \cdot v_{\mathrm{R}} = 0. \tag{15}$$

Here we consider only the solutions with $|v_{\mathrm{R}}| \neq 0$. Eliminating $v_{\mathrm{R}} \cdot \overline{v_{\mathrm{R}}}$, ϕ_1 and ϕ_2 from (12)–(15), one obtains a fourth-order equation in ϕ_3,

$$\left(\phi_3 + \frac{\mathcal{M}_2}{10} \right) \left\{ 8 \, \phi_3^3 - 15 \, \mathcal{M}_1 \phi_3^2 + 14 \, \mathcal{M}_1^2 \phi_3 - 3 \, \mathcal{M}_1^3 \\ + (\phi_3 - \mathcal{M}_1)^2 \, \mathcal{M}_2 \right\} = 0, \tag{16}$$

where

$$\mathcal{M}_1 \equiv 12 \left(\frac{m_1}{\lambda_1} \right), \, \mathcal{M}_2 \equiv 60 \left(\frac{m_2}{\lambda_2} \right). \tag{17}$$

Any solution of the cubic equation in ϕ_3 is accompanied by the solutions

$$\begin{aligned} \phi_1 &= -\frac{\phi_3}{\sqrt{6}} \frac{\left(\mathcal{M}_1^2 - 5 \, \phi_3^2\right)}{(\mathcal{M}_1 - \phi_3)^2}, \\ \phi_2 &= -\frac{1}{\sqrt{2}} \frac{\left(\mathcal{M}_1^2 - 2 \, \mathcal{M}_1 \phi_3 - \phi_3^2\right)}{(\mathcal{M}_1 - \phi_3)}, \\ v_{\mathrm{R}} \cdot \overline{v_{\mathrm{R}}} &= \frac{5}{3} \left(\frac{\lambda_1}{\lambda_2} \right) \frac{\phi_3 \, (\mathcal{M}_1 - 3 \, \phi_3) \left(\mathcal{M}_1^2 + \phi_3^2\right)}{(\mathcal{M}_1 - \phi_3)^2}. \end{aligned} \tag{18}$$

The linear term gives the solution of the fourth-order equation (16) which is very simple, $\phi_3 = -6 \left(\frac{m_2}{\lambda_2} \right)$. It leads to $\phi_1 = -\sqrt{6} \left(\frac{m_2}{\lambda_2} \right)$, $\phi_2 = -3\sqrt{2} \left(\frac{m_2}{\lambda_2} \right)$ and $\sqrt{(v_{\mathrm{R}} \cdot \overline{v_{\mathrm{R}}})} = \sqrt{60} \left(\frac{m_2}{\lambda_2} \right) \sqrt{2 \left(\frac{m_1}{m_2} \right) - 3 \left(\frac{\lambda_1}{\lambda_2} \right)}$. This solution preserves the SU(5) symmetry. Therefore, it is physically not interesting. The cubic term solutions lead to the true $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_Y$ symmetry.

4 Would-be NG bosons

In order to check the number of NG modes we write down the mass matrices for the Higgs(ino) fields which transmute the non-MSSM SO(10) gauge fields into very massive gauge fields. At first, we list the quantum numbers of the would-be NG modes under $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_Y$:
(1) $\left[\left(\overline{\mathbf{3}}, \mathbf{2}, \frac{5}{6} \right) \oplus \left(\mathbf{3}, \mathbf{2}, -\frac{5}{6} \right) \right]$,
(2) $\left[\left(\overline{\mathbf{3}}, \mathbf{2}, -\frac{1}{6} \right) \oplus \left(\mathbf{3}, \mathbf{2}, \frac{1}{6} \right) \right]$,
(3) $\left[\left(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3} \right) \oplus \left(\mathbf{3}, \mathbf{1}, \frac{2}{3} \right) \right]$,
(4) $\left[(\mathbf{1}, \mathbf{1}, 1) \oplus (\mathbf{1}, \mathbf{1}, -1) \right]$,
(5) $\left[(\mathbf{1}, \mathbf{1}, 0) \right]$.

The total number of NG degrees of freedom is $12 + 12 + 6 + 2 + 1 = 33$. In the following subsections we give explicit expressions for the mass matrices and check that their determinants are zero. The mass matrices receive contributions from the F terms in the Higgs potential. The matrix elements of the mass matrices comprise the CG coefficients which appear as coefficients of the triple products of the $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_Y$ components of the Higgs superfields. For the calculation of the CG coefficients, one must first find the explicit expressions for the $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_Y$ components of the Higgs superfields. The complete tables of the CG coefficients of a more general Higgs sector are given in a separate publication [17] and we will list only the mass matrices in this paper.

Note that the mass matrix for every irreducible representation under $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_Y$ with $Y \neq 0$ and the mass matrix for the corresponding complex conjugate representation are equal up to transposition. Therefore, only one of the two accompanied mass matrices is listed.

Of course, when enumerating the total degrees of freedom, one has to be careful to include all the mass eigenvalues (472 in total). The mass matrices define the mass part of the superpotential as a bilinear form of the fields and corresponding complex conjugate fields. The basis for the mass matrix is defined as a row of the fields multiplying the mass matrix form the left.

4.1 $[(\overline{\mathbf{3}},\mathbf{2},\tfrac{\mathbf{5}}{\mathbf{6}})\oplus(\mathbf{3},\mathbf{2},-\tfrac{\mathbf{5}}{\mathbf{6}})]$

In the basis $\left\{\Phi^{(\mathbf{3},\mathbf{2},-\frac{5}{6})}_{(\mathbf{6},\mathbf{2},\mathbf{2})},\Phi^{(\mathbf{3},\mathbf{2},-\frac{5}{6})}_{(\mathbf{10},\mathbf{2},\mathbf{2})}\right\}$ (here and hereafter the lower indices indicate $\mathrm{SU}(4)_C\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{SU}(2)_\mathrm{R}$ and the upper $\mathrm{SU}(3)_C\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_Y$ in the case of double indices), the mass matrix is written as

$$\begin{pmatrix} 2m_1-\frac{\lambda_1\phi_3}{6} & \frac{\lambda_1\phi_3}{3\sqrt{2}} \\ \frac{\lambda_1\phi_3}{3\sqrt{2}} & 2m_1+\frac{\lambda_1\phi_2}{3\sqrt{2}}-\frac{\lambda_1\phi_3}{6} \end{pmatrix}. \tag{19}$$

The determinant is indeed zero assuming the VEV conditions, (12)–(15).

4.2 $[(\overline{\mathbf{3}},\mathbf{2},-\tfrac{\mathbf{1}}{\mathbf{6}})\oplus(\mathbf{3},\mathbf{2},\tfrac{\mathbf{1}}{\mathbf{6}})]$

In the basis $\left\{\Phi^{(\mathbf{3},\mathbf{2},\frac{1}{6})}_{(\mathbf{6},\mathbf{2},\mathbf{2})},\Phi^{(\mathbf{3},\mathbf{2},\frac{1}{6})}_{(\mathbf{10},\mathbf{2},\mathbf{2})},\Delta^{(\mathbf{3},\mathbf{2},\frac{1}{6})}_{(\mathbf{15},\mathbf{2},\mathbf{2})},\overline{\Delta}^{(\mathbf{3},\mathbf{2},\frac{1}{6})}_{(\mathbf{15},\mathbf{2},\mathbf{2})}\right\}$, the mass matrix is written as

$$\begin{pmatrix} 2m_1+\frac{\lambda_1\phi_3}{6} & \frac{\lambda_1\phi_3}{3\sqrt{2}} & 0 & -\frac{\lambda_2 v_\mathrm{R}}{10\sqrt{3}} \\ \frac{\lambda_1\phi_3}{3\sqrt{2}} & 2m_1+\frac{\lambda_1\phi_2}{3\sqrt{2}}+\frac{\lambda_1\phi_3}{6} & 0 & -\frac{\lambda_2 v_\mathrm{R}}{5\sqrt{6}} \\ -\frac{\lambda_2\overline{v_\mathrm{R}}}{10\sqrt{3}} & -\frac{\lambda_2\overline{v_\mathrm{R}}}{5\sqrt{6}} & 0 & m_2+\frac{\lambda_2\phi_2}{30\sqrt{2}}+\frac{\lambda_2\phi_3}{20} \\ 0 & 0 & m_2+\frac{\lambda_2\phi_2}{30\sqrt{2}}+\frac{\lambda_2\phi_3}{60} & 0 \end{pmatrix}. \tag{20}$$

The determinant is also equal zero assuming the VEV conditions.

4.3 $[(\overline{\mathbf{3}},\mathbf{1},-\tfrac{\mathbf{2}}{\mathbf{3}})\oplus(\mathbf{3},\mathbf{1},\tfrac{\mathbf{2}}{\mathbf{3}})]$

In the basis $\left\{\Phi^{(\mathbf{3},\mathbf{1},\frac{2}{3})}_{(\mathbf{15},\mathbf{1},\mathbf{1})},\Phi^{(\mathbf{3},\mathbf{1},\frac{2}{3})}_{(\mathbf{15},\mathbf{1},\mathbf{3})},\overline{\Delta}^{(\mathbf{3},\mathbf{1},\frac{2}{3})}_{(\mathbf{10},\mathbf{1},\mathbf{3})}\right\}$, the mass matrix is written as

$$\begin{pmatrix} 2m_1+\frac{\lambda_1\phi_2}{3\sqrt{2}} & \frac{\lambda_1\phi_3}{3\sqrt{2}} & -\frac{\lambda_2\overline{v_\mathrm{R}}}{10\sqrt{3}} \\ \frac{\lambda_1\phi_3}{3\sqrt{2}} & 2m_1+\frac{\lambda_1\phi_1}{\sqrt{6}}+\frac{\lambda_1\phi_2}{3\sqrt{2}} & -\frac{\lambda_2\overline{v_\mathrm{R}}}{5\sqrt{6}} \\ -\frac{\lambda_2 v_\mathrm{R}}{10\sqrt{3}} & -\frac{\lambda_2 v_\mathrm{R}}{5\sqrt{6}} & m_2+\frac{\lambda_2\phi_1}{10\sqrt{6}}+\frac{\lambda_2\phi_2}{30\sqrt{2}}+\frac{\lambda_2\phi_3}{30} \end{pmatrix}. \tag{21}$$

The determinant is also equal zero assuming the VEV conditions.

4.4 $[(\mathbf{1},\mathbf{1},\mathbf{1})\oplus(\mathbf{1},\mathbf{1},-\mathbf{1})]$

In the basis $\left\{\Phi^{(\mathbf{1},\mathbf{1},1)}_{(\mathbf{15},\mathbf{1},\mathbf{3})},\Delta^{(\mathbf{1},\mathbf{1},1)}_{(\overline{\mathbf{10}},\mathbf{1},3)}\right\}$, the mass matrix is written as

$$\begin{pmatrix} 2m_1+\frac{\lambda_1\phi_1}{\sqrt{6}}+\frac{\sqrt{2}\lambda_1\phi_2}{3} & -\frac{\lambda_2 v_\mathrm{R}}{10} \\ -\frac{\lambda_2\overline{v_\mathrm{R}}}{10} & m_2+\frac{\lambda_2\phi_1}{10\sqrt{6}}+\frac{\lambda_2\phi_2}{10\sqrt{2}} \end{pmatrix}. \tag{22}$$

The determinant is also equal zero assuming the VEV conditions.

4.5 $[(\mathbf{1},\mathbf{1},\mathbf{0})]$

In the basis $\left\{\Phi^{(\mathbf{1},\mathbf{1},0)}_{(\mathbf{1},\mathbf{1},\mathbf{1})},\Phi^{(\mathbf{1},\mathbf{1},0)}_{(\mathbf{15},\mathbf{1},\mathbf{1})},\Phi^{(\mathbf{1},\mathbf{1},0)}_{(\mathbf{15},\mathbf{1},\mathbf{3})},\overline{\Delta}^{(\mathbf{1},\mathbf{1},0)}_{(\mathbf{10},\mathbf{1},\mathbf{3})},\Delta^{(\mathbf{1},\mathbf{1},0)}_{(\overline{\mathbf{10}},\mathbf{1},\mathbf{3})}\right\}$, the mass matrix is written as

$$\begin{pmatrix} 2m_1 & 0 & \frac{\lambda_1\phi_3}{\sqrt{6}} & -\frac{\lambda_2\overline{v_\mathrm{R}}}{10\sqrt{6}} & -\frac{\lambda_2 v_\mathrm{R}}{10\sqrt{6}} \\ 0 & 2m_1+\frac{\sqrt{2}\lambda_1\phi_2}{3} & \frac{\sqrt{2}\lambda_1\phi_3}{3} & -\frac{\lambda_2\overline{v_\mathrm{R}}}{10\sqrt{2}} & -\frac{\lambda_2 v_\mathrm{R}}{10\sqrt{2}} \\ \frac{\lambda_1\phi_3}{\sqrt{6}} & \frac{\sqrt{2}\lambda_1\phi_3}{3} & 2m_1+\frac{\lambda_1\phi_1}{\sqrt{6}}+\frac{\sqrt{2}\lambda_1\phi_2}{3} & -\frac{\lambda_2\overline{v_\mathrm{R}}}{10} & -\frac{\lambda_2 v_\mathrm{R}}{10} \\ -\frac{\lambda_2 v_\mathrm{R}}{10\sqrt{6}} & -\frac{\lambda_2 v_\mathrm{R}}{10\sqrt{2}} & -\frac{\lambda_2 v_\mathrm{R}}{10} & e4 & 0 \\ -\frac{\lambda_2\overline{v_\mathrm{R}}}{10\sqrt{6}} & -\frac{\lambda_2\overline{v_\mathrm{R}}}{10\sqrt{2}} & -\frac{\lambda_2\overline{v_\mathrm{R}}}{10} & 0 & e4 \end{pmatrix}. \tag{23}$$

Here $e4\equiv m_2+\lambda_2\left(\frac{\phi_1}{10\sqrt{6}}+\frac{\phi_2}{10\sqrt{2}}+\frac{\phi_3}{10}\right)$ is nothing but the left-hand side of (15) divided by v_R, and (23) has one zero eigenvalue.

5 Electroweak Higgs doublet

In the standard picture of the electroweak symmetry breaking, we have the Higgs doublets which give masses to the matter. These masses should be less than or equal to the electroweak scale. Since we approximate the electroweak scale by zero, we must impose a constraint that the mass matrix should have one zero eigenvalue.

We define

$$H^{10}_u\equiv H^{(\mathbf{1},\mathbf{2},\frac{1}{2})}_{(\mathbf{1},\mathbf{2},\mathbf{2})},\ \overline{\Delta}_u\equiv\overline{\Delta}^{(\mathbf{1},\mathbf{2},\frac{1}{2})}_{(\mathbf{15},\mathbf{2},\mathbf{2})},$$
$$\Delta_u\equiv\Delta^{(\mathbf{1},\mathbf{2},\frac{1}{2})}_{(\mathbf{15},\mathbf{2},\mathbf{2})},\ \Phi_u\equiv\Phi^{(\mathbf{1},\mathbf{2},\frac{1}{2})}_{(\overline{\mathbf{10}},2,2)}. \tag{24}$$

and

$$H^{10}_d\equiv H^{(\mathbf{1},\mathbf{2},-\frac{1}{2})}_{(\mathbf{1},\mathbf{2},\mathbf{2})},\ \overline{\Delta}_d\equiv\overline{\Delta}^{(\mathbf{1},\mathbf{2},-\frac{1}{2})}_{(\mathbf{15},\mathbf{2},\mathbf{2})},$$
$$\Delta_d\equiv\Delta^{(\mathbf{1},\mathbf{2},-\frac{1}{2})}_{(\mathbf{15},\mathbf{2},\mathbf{2})},\ \Phi_d\equiv\Phi^{(\mathbf{1},\mathbf{2},-\frac{1}{2})}_{(\mathbf{10},\mathbf{2},\mathbf{2})}. \tag{25}$$

In the basis $\left\{H_u^{10}, \overline{\Delta}_u, \Delta_u, \Phi_u\right\}$, the mass matrix is written as

$$
M_{\text{doublet}} \equiv \begin{pmatrix} 2m_3 & \frac{\lambda_3\phi_2}{\sqrt{10}} - \frac{\lambda_3\phi_3}{2\sqrt{5}} & -\frac{\lambda_4\phi_2}{\sqrt{10}} - \frac{\lambda_4\phi_3}{2\sqrt{5}} & \frac{\lambda_4\overline{v}_{\rm R}}{\sqrt{5}} \\ \frac{\lambda_4\phi_2}{\sqrt{10}} - \frac{\lambda_4\phi_3}{2\sqrt{5}} & m_2 + \frac{\lambda_2\phi_2}{15\sqrt{2}} - \frac{\lambda_2\phi_3}{30} & 0 & 0 \\ -\frac{\lambda_3\phi_2}{\sqrt{10}} - \frac{\lambda_3\phi_3}{2\sqrt{5}} & 0 & m_2 + \frac{\lambda_2\phi_2}{15\sqrt{2}} + \frac{\lambda_2\phi_3}{30} & -\frac{\lambda_2\overline{v}_{\rm R}}{10} \\ \frac{\lambda_3 v_{\rm R}}{\sqrt{5}} & 0 & -\frac{\lambda_2 v_{\rm R}}{10} & 2m_1 + \frac{\lambda_1\phi_2}{\sqrt{2}} + \frac{\lambda_1\phi_3}{2} \end{pmatrix}. \tag{26}
$$

The corresponding mass terms of the superpotential read

$$
W_m = \left(H_u^{10}, \overline{\Delta}_u, \Delta_u, \Phi_u\right) M_{\text{doublet}} \left(H_d^{10}, \Delta_d, \overline{\Delta}_d, \Phi_d\right)^{\rm T}. \tag{27}
$$

The requirement of the existence of a zero mode leads to the following condition:

$$
\det M_{\text{doublet}} = 0. \tag{28}
$$

For instance, in case of $\lambda_3 = 0$, $m_2 + \frac{\lambda_2\phi_2}{15\sqrt{2}} - \frac{\lambda_2\phi_3}{30} = 0$, we obtain a special solution to (28), while it keeps a desirable vacuum and it does not produce any additional massless fields. For instance, for $\phi_1/\mathcal{M}_1 = -0.06077$, $\phi_2/\mathcal{M}_1 = -0.5949$, $\phi_3/\mathcal{M}_1 = 0.1238$, $v_{\rm R}\overline{v}_{\rm R}/\mathcal{M}_1^2 = 0.1715\lambda_1/\lambda_2$ with the condition $\mathcal{M}_2/\mathcal{M}_1 = 1.930$, both the condition below (28) and (16) are satisfied. However, we proceed our arguments hereafter without using this special solution.

We can diagonalize the mass matrix, M_{doublet}, by a bi-unitary transformation.

$$
U^* M_{\text{doublet}} V^\dagger = \operatorname{diag}(0, M_1, M_2, M_3). \tag{29}
$$

Then the mass eigenstates are written as

$$
\begin{aligned}
\left(H_u, \mathrm{h}_u^1, \mathrm{h}_u^2, \mathrm{h}_u^3\right) &= \left(H_u^{10}, \overline{\Delta}_u, \Delta_u, \Phi_u\right) U^{\rm T}, \\
\left(H_d, \mathrm{h}_d^1, \mathrm{h}_d^2, \mathrm{h}_d^3\right) &= \left(H_d^{10}, \Delta_d, \overline{\Delta}_d, \Phi_d\right) V^{\rm T}.
\end{aligned} \tag{30}
$$

The representations **45** and/or **54**, and higher dimensional operators, are not included in our minimal model. Therefore, we must set the "doublet–triplet splittings" by hand as (28).

By making the inverse transformation of (30), the following expressions are obtained:

$$
\begin{aligned}
H_u^{10} &= \alpha_u H_u + \cdots, \; H_d^{10} = \alpha_d H_d + \cdots, \\
\overline{\Delta}_u &= \beta_u H_u + \cdots, \; \overline{\Delta}_d = \beta_d H_d + \cdots,
\end{aligned} \tag{31}
$$

where "$+\cdots$" represent the heavy Higgs fields, $\mathrm{h}_{u,d}^i$ ($i = 1, 2, 3$) which are integrated out when considering the low-energy effective superpotential.

Precisely, we can read off from (30) that

$$
\begin{aligned}
\alpha_u &= (U^*)_{11}, \; \beta_u = (U^*)_{12}, \\
\alpha_d &= (V^*)_{11}, \; \beta_d = (V^*)_{13}.
\end{aligned} \tag{32}
$$

Using the two pairs of Higgs doublets, $H_{u,d}^{10}$ and $\overline{\Delta}_{u,d}$, the Yukawa couplings of (2) are rewritten as

$$
\begin{aligned}
W_Y = {} & u_i^c \left(Y_{10}^{ij} H_u^{10} + Y_{126}^{ij} \overline{\Delta}_u\right) q_j \\
& + d_i^c \left(Y_{10}^{ij} H_d^{10} + Y_{126}^{ij} \overline{\Delta}_d\right) q_j \\
& + \nu_i^c \left(Y_{10}^{ij} H_u^{10} - 3Y_{126}^{ij} \overline{\Delta}_u\right) \ell_j \\
& + e_i^c \left(Y_{10}^{ij} H_d^{10} - 3Y_{126}^{ij} \overline{\Delta}_d\right) \ell_j \\
& + \nu_i^c \left(Y_{126}^{ij} v_{\rm R}\right) \nu_j^c.
\end{aligned} \tag{33}
$$

By using (31), we obtain the low-energy effective superpotential which is described by only the light Higgs doublets H_u and H_d,

$$
\begin{aligned}
W_{\text{eff}} = {} & u_i^c \left(\alpha_u Y_{10}^{ij} + \beta_u Y_{126}^{ij}\right) H_u \, q_j \\
& + d_i^c \left(\alpha_d Y_{10}^{ij} + \beta_d Y_{126}^{ij}\right) H_d \, q_j \\
& + \nu_i^c \left(\alpha_u Y_{10}^{ij} - 3\beta_u Y_{126}^{ij}\right) H_u \, \ell_j \\
& + e_i^c \left(\alpha_d Y_{10}^{ij} - 3\beta_d Y_{126}^{ij}\right) H_d \, \ell_j \\
& + \nu_i^c \left(Y_{126}^{ij} v_{\rm R}\right) \nu_j^c + \mu_{\text{eff}} \, H_u H_d.
\end{aligned} \tag{34}
$$

Here we have assumed that some mechanism, like the Giudice–Masiero mechanism [18] in supergravity, may produce the effective μ term, μ_{eff}, for the light Higgs doublets.

6 Proton decay

After the symmetry breaking from SO(10) to $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_{\rm L} \times \mathrm{U}(1)_Y$, the generic Yukawa interactions between the matter fields and the color triplet Higgs fields are given by

$$
\begin{aligned}
W_Y = {} & Y_{10}^{ij} H_{\overline{T}} \left(q_i \ell_j + u_i^c d_j^c\right) + Y_{126}^{ij} \overline{\Delta}_{\overline{T}} \left(q_i \ell_j + u_i^c d_j^c\right) \\
& + Y_{10}^{ij} H_T \left(\frac{1}{2} q_i q_j + u_i^c e_j^c + d_i^c \nu_j^c\right) \\
& + Y_{126}^{ij} \overline{\Delta}_T \left(\frac{1}{2} q_i q_j + u_i^c e_j^c + d_i^c \nu_j^c\right) \\
& + Y_{126}^{ij} \overline{\Delta}'_T \left(u_i^c e_j^c + d_i^c \nu_j^c\right).
\end{aligned} \tag{35}
$$

Here we have defined

$$
\begin{aligned}
& H_{\overline{T}} \equiv H_{(\mathbf{6},\mathbf{1},\mathbf{1})}^{(\overline{\mathbf{3}},1,\frac{1}{3})}, \; H_T \equiv H_{(\mathbf{6},\mathbf{1},\mathbf{1})}^{(\mathbf{3},\mathbf{1},-\frac{1}{3})}, \; \overline{\Delta}_{\overline{T}} \equiv \overline{\Delta}_{(\mathbf{6},\mathbf{1},\mathbf{1})}^{(\overline{\mathbf{3}},1,\frac{1}{3})}, \\
& \overline{\Delta}_T \equiv \overline{\Delta}_{(\mathbf{6},\mathbf{1},\mathbf{1})}^{(\mathbf{3},\mathbf{1},-\frac{1}{3})}, \; \overline{\Delta}'_T \equiv \overline{\Delta}_{(\mathbf{10},\mathbf{1},\mathbf{3})}^{(\mathbf{3},\mathbf{1},-\frac{1}{3})}.
\end{aligned} \tag{36}
$$

For later use we define

$$
\begin{aligned}
& \Delta_{\overline{T}} \equiv \Delta_{(\mathbf{6},\mathbf{1},\mathbf{1})}^{(\overline{\mathbf{3}},1,\frac{1}{3})}, \; \Delta_T \equiv \Delta_{(\mathbf{6},\mathbf{1},\mathbf{1})}^{(\mathbf{3},\mathbf{1},-\frac{1}{3})}, \; \Delta'_{\overline{T}} \equiv \Delta_{(\overline{\mathbf{10}},1,3)}^{(\overline{\mathbf{3}},1,\frac{1}{3})}, \\
& \Phi_{\overline{T}} \equiv \Phi_{(\mathbf{15},\mathbf{1},\mathbf{3})}^{(\overline{\mathbf{3}},1,\frac{1}{3})}, \; \Phi_T \equiv \Phi_{(\mathbf{15},\mathbf{1},\mathbf{3})}^{(\mathbf{3},\mathbf{1},-\frac{1}{3})}.
\end{aligned} \tag{37}
$$

In the basis $\left\{H_{\overline{T}}, \Delta_{\overline{T}}, \overline{\Delta}_{\overline{T}}, \Phi_{\overline{T}}, \Delta'_{\overline{T}}\right\}$, the mass matrix reads

$$
M_{\text{triplet}} \equiv \begin{pmatrix}
2m_3 & -\frac{\lambda_4\phi_1}{\sqrt{10}} - \frac{\lambda_4\phi_2}{\sqrt{30}} & -\frac{\lambda_3\phi_1}{\sqrt{10}} + \frac{\lambda_3\phi_2}{\sqrt{30}} & \frac{\lambda_4\overline{v_{\mathrm{R}}}}{\sqrt{5}} & -\frac{\sqrt{2}\lambda_4\phi_3}{\sqrt{15}} \\
-\frac{\lambda_3\phi_1}{\sqrt{10}} - \frac{\lambda_3\phi_2}{\sqrt{30}} & m_2 & 0 & -\frac{\lambda_2\overline{v_{\mathrm{R}}}}{10\sqrt{3}} & \frac{\lambda_2\phi_3}{15\sqrt{2}} \\
-\frac{\lambda_4\phi_1}{\sqrt{10}} + \frac{\lambda_4\phi_2}{\sqrt{30}} & 0 & m_2 & 0 & 0 \\
\frac{\lambda_3 v_{\mathrm{R}}}{\sqrt{5}} & -\frac{\lambda_2 v_{\mathrm{R}}}{10\sqrt{3}} & 0 & \overline{m}_{44} & -\frac{\lambda_2 v_{\mathrm{R}}}{5\sqrt{6}} \\
-\frac{\sqrt{2}\lambda_3\phi_3}{\sqrt{15}} & \frac{\lambda_2\phi_3}{15\sqrt{2}} & 0 & -\frac{\lambda_2\overline{v_{\mathrm{R}}}}{5\sqrt{6}} & \overline{m}_{55}
\end{pmatrix}, \tag{38}
$$

where $\overline{m}_{44} \equiv 2m_1 + \frac{\lambda_1\phi_1}{\sqrt{6}} + \frac{\lambda_1\phi_2}{3\sqrt{2}} + \frac{2\lambda_1\phi_3}{3}$ and $\overline{m}_{55} \equiv m_2 + \frac{\lambda_2\phi_1}{10\sqrt{6}} + \frac{\lambda_2\phi_2}{30\sqrt{2}}$. The corresponding mass terms of the superpotential read

$$
\begin{aligned}
W_m = &\left(H_{\overline{T}}, \Delta_{\overline{T}}, \overline{\Delta}_{\overline{T}}, \Phi_{\overline{T}}, \Delta'_{\overline{T}}\right) \\
&\times M_{\text{triplet}} \left(H_T, \overline{\Delta}_T, \Delta_T, \Phi_T, \overline{\Delta}'_T\right)^{\mathrm{T}}.
\end{aligned} \tag{39}
$$

Now we integrate out all the color triplet Higgs fields in order to obtain the effective dimension-five operators related to the proton decay.[2] We first integrate out the color triplet Higgs fields, Δ_T and Φ_T,

$$
\begin{bmatrix} \overline{\Delta}_{\overline{T}} \\ \Phi_{\overline{T}} \end{bmatrix} = -\frac{1}{\mathcal{D}} \times \begin{bmatrix} \left(2m_1 + \frac{\lambda_1\phi_1}{\sqrt{6}} + \frac{\lambda_1\phi_2}{3\sqrt{2}} + \frac{2\lambda_1\phi_3}{3}\right)\left(-\frac{\lambda_3\phi_1}{\sqrt{10}} + \frac{\lambda_3\phi_2}{\sqrt{30}}\right) H_{\overline{T}} \\ m_2 \cdot \frac{\lambda_4\overline{v_{\mathrm{R}}}}{\sqrt{5}} H_{\overline{T}} - m_2 \cdot \frac{\lambda_2\overline{v_{\mathrm{R}}}}{10\sqrt{3}} \Delta_{\overline{T}} - m_2 \cdot \frac{\lambda_2\overline{v_{\mathrm{R}}}}{5\sqrt{6}} \Delta'_{\overline{T}} \end{bmatrix},
$$

where

$$
\mathcal{D} \equiv m_2 \cdot \left(2m_1 + \frac{\lambda_1\phi_1}{\sqrt{6}} + \frac{\lambda_1\phi_2}{3\sqrt{2}} + \frac{2\lambda_1\phi_3}{3}\right). \tag{40}
$$

Putting this into the original mass terms of the superpotential (39), we can obtain the following mass terms for the color triplet Higgs fields:

$$
W_m^{\text{eff}} = \left(H_{\overline{T}}, \Delta_{\overline{T}}, \Delta'_{\overline{T}}\right) M_{\text{triplet}}^{\text{eff}} \left(H_T, \overline{\Delta}_T, \overline{\Delta}'_T\right)^{\mathrm{T}}. \tag{41}
$$

Here the explicit forms of the elements of this mass matrix, $M_{\text{triplet}}^{\text{eff}} = \{m_{ij}\}$, are given as follows:

$$
\begin{aligned}
m_{11} \equiv\ & 2m_3 - \frac{1}{\mathcal{D}}\left[\left(-\frac{\lambda_4\phi_1}{\sqrt{10}} + \frac{\lambda_4\phi_2}{\sqrt{30}}\right)\right. \\
&\times \left(2m_1 + \frac{\lambda_1\phi_1}{\sqrt{6}} + \frac{\lambda_1\phi_2}{3\sqrt{2}} + \frac{2\lambda_1\phi_3}{3}\right) \\
&\times \left(-\frac{\lambda_3\phi_1}{\sqrt{10}} + \frac{\lambda_3\phi_2}{\sqrt{30}}\right) \\
&\left. + \frac{\lambda_3 v_{\mathrm{R}}}{\sqrt{5}} \cdot m_2 \cdot \frac{\lambda_4\overline{v_{\mathrm{R}}}}{\sqrt{5}}\right], \\
m_{12} \equiv\ & -\frac{\lambda_4\phi_1}{\sqrt{10}} - \frac{\lambda_4\phi_2}{\sqrt{30}} + \frac{1}{\mathcal{D}} \frac{\lambda_2 v_{\mathrm{R}}}{10\sqrt{3}} \cdot m_2 \cdot \frac{\lambda_4\overline{v_{\mathrm{R}}}}{\sqrt{5}}, \\
m_{13} \equiv\ & -\frac{\sqrt{2}\lambda_4\phi_3}{\sqrt{15}} + \frac{1}{\mathcal{D}} \frac{\lambda_2 v_{\mathrm{R}}}{5\sqrt{6}} \cdot m_2 \cdot \frac{\lambda_4\overline{v_{\mathrm{R}}}}{\sqrt{5}}, \\
m_{21} \equiv\ & -\frac{\lambda_3\phi_1}{\sqrt{10}} - \frac{\lambda_3\phi_2}{\sqrt{30}} + \frac{1}{\mathcal{D}} \frac{\lambda_3 v_{\mathrm{R}}}{\sqrt{5}} \cdot m_2 \cdot \frac{\lambda_2\overline{v_{\mathrm{R}}}}{10\sqrt{3}}, \\
m_{22} \equiv\ & m_2 - \frac{1}{\mathcal{D}} \frac{\lambda_2 v_{\mathrm{R}}}{10\sqrt{3}} \cdot m_2 \cdot \frac{\lambda_2\overline{v_{\mathrm{R}}}}{10\sqrt{3}}, \\
m_{23} \equiv\ & \frac{\lambda_2\phi_3}{15\sqrt{2}} - \frac{1}{\mathcal{D}} \frac{\lambda_2 v_{\mathrm{R}}}{10\sqrt{3}} \cdot m_2 \cdot \frac{\lambda_2\overline{v_{\mathrm{R}}}}{10\sqrt{3}}, \\
m_{31} \equiv\ & -\frac{\sqrt{2}\lambda_3\phi_3}{\sqrt{15}} + \frac{1}{\mathcal{D}} \frac{\lambda_3 v_{\mathrm{R}}}{\sqrt{5}} \cdot m_2 \cdot \frac{\lambda_2\overline{v_{\mathrm{R}}}}{5\sqrt{6}}, \\
m_{32} \equiv\ & \frac{\lambda_2\phi_3}{15\sqrt{2}} - \frac{1}{\mathcal{D}} \frac{\lambda_2 v_{\mathrm{R}}}{10\sqrt{3}} \cdot m_2 \cdot \frac{\lambda_2\overline{v_{\mathrm{R}}}}{5\sqrt{6}}, \\
m_{33} \equiv\ & m_2 + \frac{\lambda_2\phi_1}{10\sqrt{6}} + \frac{\lambda_2\phi_2}{30\sqrt{2}} \\
& - \frac{1}{\mathcal{D}} \frac{\lambda_2 v_{\mathrm{R}}}{5\sqrt{6}} \cdot m_2 \cdot \frac{\lambda_2\overline{v_{\mathrm{R}}}}{5\sqrt{6}}.
\end{aligned} \tag{42}
$$

Moreover, integrating out the color triplet Higgs field $\Delta'_{\overline{T}}$, we obtain the effective Yukawa interactions between the matter fields and the color triplet Higgs fields as

$$
\begin{aligned}
W_Y =\ & Y_{10}^{ij}\, H_{\overline{T}} \left(q_i \ell_j + u_i^c d_j^c\right) + Y_{126}^{ij}\, \overline{\Delta}_{\overline{T}} \left(q_i \ell_j + u_i^c d_j^c\right) \\
& + Y_{10}^{ij}\, H_T\, \frac{1}{2} q_i q_j \\
& + \left(Y_{10}^{ij} - \frac{m_{31}}{m_{33}} Y_{126}^{ij}\right) H_T \left(u_i^c e_j^c + d_i^c \nu_j^c\right) \\
& + Y_{126}^{ij}\, \overline{\Delta}_T\, \frac{1}{2} q_i q_j \\
& + \left(1 - \frac{m_{32}}{m_{33}}\right) Y_{126}^{ij}\, \overline{\Delta}_T \left(u_i^c e_j^c + d_i^c \nu_j^c\right).
\end{aligned} \tag{43}
$$

Then the effective mass terms for the remaining color triplet Higgs fields are written as

$$
\begin{aligned}
W_m^{\text{eff}} &= H_{\overline{T}} \left(a\, H_T + b\, \overline{\Delta}_T\right) + \overline{\Delta}_{\overline{T}} \left(c\, H_T + d\, \overline{\Delta}_T\right) \\
&\equiv \left(H_{\overline{T}}, \overline{\Delta}_{\overline{T}}\right) M_T \begin{pmatrix} H_T \\ \overline{\Delta}_T \end{pmatrix},
\end{aligned} \tag{44}
$$

where a, b, c, d are defined by

$$
\begin{aligned}
a &\equiv m_{11} - \frac{m_{13}}{m_{33}} \cdot m_{31}, \quad b \equiv m_{12} - \frac{m_{13}}{m_{33}} \cdot m_{32}, \\
c &\equiv m_{21} - \frac{m_{23}}{m_{33}} \cdot m_{31}, \quad d \equiv m_{22} - \frac{m_{23}}{m_{33}} \cdot m_{32}.
\end{aligned} \tag{45}
$$

Combining (43) and (44) leads to the effective dimension-five interactions after integrating out the remaining color

[2] The integration procedure presented here is equivalent to the integration procedure after the diagonalization of the full triplet 5×5 matrix.

triplet Higgs fields [19],

$$-W_5 = C_{\mathrm{L}}^{ijkl}\,\frac{1}{2}q_iq_jq_k\ell_l + C_{\mathrm{R}}^{ijkl}\,u_i^ce_j^cu_k^cd_l^c, \tag{46}$$

inducing the dangerous proton decay. Here, C_{L} and C_{R} are given by the Yukawa coupling matrices at the GUT scale, M_G,

$$\begin{aligned} C_{\mathrm{L}}^{ijkl}(M_G) &= \left(Y_{10}^{ij},\,Y_{126}^{ij}\right)M_T^{-1}\begin{pmatrix}Y_{10}^{kl}\\ Y_{126}^{kl}\end{pmatrix},\\ C_{\mathrm{R}}^{ijkl}(M_G) &= \left(Y_{10}^{ij}-\frac{m_{13}}{m_{33}}\,Y_{126}^{ij},\,\left(1-\frac{m_{32}}{m_{33}}\right)Y_{126}^{ij}\right)M_T^{-1}\\ &\quad\times\begin{pmatrix}Y_{10}^{kl}\\ Y_{126}^{kl}\end{pmatrix}. \end{aligned}\tag{47}$$

Note that

$$\begin{aligned}\begin{pmatrix}Y_{10}\\ Y_{126}\end{pmatrix} &= \begin{pmatrix}\alpha_u & \beta_u\\ \alpha_d & \beta_d\end{pmatrix}^{-1}\begin{pmatrix}Y_u\\ Y_d\end{pmatrix}\\ &\equiv A^{-1}\begin{pmatrix}Y_u\\ Y_d\end{pmatrix}.\end{aligned}\tag{48}$$

Thus we have

$$C_{\mathrm{L}}^{ijkl} = \left(Y_u^{ij},\;Y_d^{ij}\right)\left(A\,M_T\,A^{\mathrm{T}}\right)^{-1}\begin{pmatrix}Y_u^{kl}\\ Y_d^{kl}\end{pmatrix}. \tag{49}$$

We make use of this expressions in order to evaluate the renormalization group effects on the Wilson coefficients C_{L}^{ijkl} and C_{R}^{ijkl}. Without loss of generality, we can use the basis where Y_u is real and diagonal,

$$Y_u = \frac{1}{v\sin\beta}\,\mathrm{diag}(m_u,m_c,m_t), \tag{50}$$

with $v \simeq 174.1\,\mathrm{GeV}$. Since Y_d is a symmetric matrix, it can be described by

$$Y_d = \frac{1}{v\cos\beta}\,\overline{V}_{\mathrm{CKM}}^{\,*}\,\mathrm{diag}(m_d,m_s,m_b)\,\overline{V}_{\mathrm{CKM}}^{\,\dagger}, \tag{51}$$

by using a unitary matrix

$$\overline{V}_{\mathrm{CKM}} \equiv \mathrm{e}^{\mathrm{i}\alpha_1}\,\mathrm{e}^{\mathrm{i}\alpha_2\lambda_3}\,\mathrm{e}^{\mathrm{i}\alpha_3\lambda_8}\,V_{\mathrm{CKM}}\,e^{\mathrm{i}\beta_2\lambda_3}\,\mathrm{e}^{\mathrm{i}\beta_3\lambda_8}, \tag{52}$$

where λ_3, λ_8 are the Gell-Mann matrices and V_{CKM} is the Cabibbo–Kobayashi–Maskawa (CKM) mixing matrix [20].[3]

The complete antisymmetry in the color indices requires that the dimension-five operator (46) possesses non-diagonal flavor indices [21]. As a consequence, the dominant decay mode is $p \to K^+\bar{\nu}$. This fact implies that the chargino dressing diagrams dominate over the gluino and the neutralino dressing diagrams [22].

In the component form, the dimension-five operators at the SUSY breaking scale, M_{SUSY}, are written

$$\begin{aligned}\mathcal{L}_5 = {} & C_{\mathrm{L}}^{(\tilde{u}\tilde{d}ue)XYij}\,\widetilde{u_X}\widetilde{d_Y}u_{\mathrm{L}i}e_{\mathrm{L}j}\\ &+C_{\mathrm{L}}^{(\tilde{u}\tilde{u}de)XYij}\,\frac{1}{2}\widetilde{u_X}\widetilde{u_Y}d_{\mathrm{L}i}e_{\mathrm{L}j}\\ &+C_{\mathrm{R}}^{(\tilde{u}\tilde{d}ue)XYij}\,\widetilde{u_X}\widetilde{d_Y}u_{\mathrm{R}i}e_{\mathrm{R}j}\\ &+C_{\mathrm{R}}^{(\tilde{u}\tilde{u}de)XYij}\,\frac{1}{2}\widetilde{u_X}\widetilde{u_Y}d_{\mathrm{R}i}e_{\mathrm{R}j}\\ &+C_{\mathrm{L}}^{(\tilde{u}\tilde{d}d\nu)XYij}\,\widetilde{u_X}\widetilde{d_Y}d_{\mathrm{L}i}\nu_{\mathrm{L}j}\\ &+C_{\mathrm{L}}^{(\tilde{d}\tilde{d}u\nu)XYij}\,\frac{1}{2}\widetilde{d_X}\widetilde{d_Y}u_{\mathrm{L}i}\nu_{\mathrm{L}j}\\ &+C_{\mathrm{L}}^{(\tilde{u}\tilde{e}ud)XYij}\,\widetilde{u_X}\widetilde{e_Y}u_{\mathrm{L}i}d_{\mathrm{L}j}\\ &+C_{\mathrm{L}}^{(\tilde{d}\tilde{e}uu)XYij}\,\frac{1}{2}\widetilde{d_X}\widetilde{e_Y}u_{\mathrm{L}i}u_{\mathrm{L}j}\\ &+C_{\mathrm{R}}^{(\tilde{u}\tilde{e}ud)XYij}\,\widetilde{u_X}\widetilde{e_Y}u_{\mathrm{R}i}d_{\mathrm{R}j}\\ &+C_{\mathrm{R}}^{(\tilde{d}\tilde{e}uu)XYij}\,\frac{1}{2}\widetilde{d_X}\widetilde{e_Y}u_{\mathrm{R}i}u_{\mathrm{R}j}\\ &+C_{\mathrm{L}}^{(\tilde{d}\tilde{\nu}ud)XYij}\,\widetilde{d_X}\widetilde{\nu_Y}u_{\mathrm{L}i}d_{\mathrm{L}j}\\ &+C_{\mathrm{L}}^{(\tilde{u}\tilde{\nu}dd)XYij}\,\frac{1}{2}\widetilde{u_X}\widetilde{\nu_Y}d_{\mathrm{L}i}d_{\mathrm{L}j}.\end{aligned}\tag{53}$$

The coefficients are obtained from the coefficients of the original dimension-five operators including their renormalization from M_G to M_{SUSY}. Their explicit forms are found in Appendix A. After the sparticle dressing, we obtain the following type of dimension-six operators causing nucleon decays:

$$\begin{aligned}\mathcal{L}_6 = \frac{1}{16\pi^2}\Big[& C_{\mathrm{LL}}^{(udue)ij}(u_{\mathrm{L}}d_{\mathrm{L}i})(u_{\mathrm{L}}e_{\mathrm{L}j})\\ &+C_{\mathrm{RL}}^{(udue)ij}(u_{\mathrm{R}}d_{\mathrm{R}i})(u_{\mathrm{L}}e_{\mathrm{L}j})\\ &+C_{\mathrm{LR}}^{(udue)ij}(u_{\mathrm{L}}d_{\mathrm{L}i})(u_{\mathrm{R}}e_{\mathrm{R}j})\\ &+C_{\mathrm{RR}}^{(udue)ij}(u_{\mathrm{R}}d_{\mathrm{R}i})(u_{\mathrm{R}}e_{\mathrm{R}j})\\ &+C_{\mathrm{LL}}^{(udd\nu)ijk}(u_{\mathrm{L}}d_{\mathrm{L}i})(d_{\mathrm{L}j}\nu_{\mathrm{L}k})\\ &+C_{\mathrm{RL}}^{(udd\nu)ijk}(u_{\mathrm{R}}d_{\mathrm{R}i})(d_{\mathrm{L}j}\nu_{\mathrm{L}k})\\ &+C_{\mathrm{RL}}^{(ddu\nu)ijk}\frac{1}{2}(d_{\mathrm{R}i}d_{\mathrm{R}j})(u_{\mathrm{L}}\nu_{\mathrm{L}k})\Big].\end{aligned}\tag{54}$$

These operators should be renormalized from M_{SUSY} to M_Z and further to the hadronization scale (μ_{had}) $\approx 1\,\mathrm{GeV}$. Then the effective four-Fermi Lagrangian is converted to a hadronic Lagrangian by using the chiral Lagrangian method [23][24]. Details are given in Appendices B and C.

For the decay mode $p \to K^+\bar{\nu}_i$, the partial decay rate is given by the formula

$$\Gamma(p\to K^+\bar{\nu}_i) = \frac{m_p}{32\pi}\left(1-\frac{m_{K^+}^2}{m_p^2}\right)^2\frac{1}{f_\pi{}^2}\,|\mathcal{A}(p\to K^+\bar{\nu}_i)|^2. \tag{55}$$

Here $m_p = 0.938\,\mathrm{GeV}$ is the proton mass, $m_{K^+} = 0.493\,\mathrm{GeV}$ is the kaon mass and $f_\pi = 0.131\,\mathrm{GeV}$ is the pion decay constant.

[3] In [6], we set these phases α_i $(i=1,2,3)$, β_i $(i=2,3)$ to zero or π.

The amplitude $\mathcal{A}(p \to K^+\bar{\nu}_i)$ for $p \to K^+\bar{\nu}_i$ reads [25]

$$\mathcal{A}(p \to K^+\bar{\nu}_i) = \left[\beta C_{\mathrm{LL}}^{(udd\nu)21i} + \alpha C_{\mathrm{RL}}^{(udd\nu)21i}\right] \frac{2m_p}{3m_B} D + \left[\beta C_{\mathrm{LL}}^{(udd\nu)12i} + \alpha C_{\mathrm{RL}}^{(udd\nu)12i}\right]\left[1 + \frac{m_p}{3m_B}(3F+D)\right] + \alpha C_{\mathrm{RL}}^{(ddu\nu)12i}\left[1 - \frac{m_p}{3m_B}(3F-D)\right]. \quad (56)$$

Here $m_B = 1.150\,\mathrm{GeV}$ is an averaged baryon mass, $F = 0.44$, $D = 0.81$ are the parameters in terms of which the octet-baryon axial-vector form factors are expressed, and α, β are the hadron matrix elements which are defined by [26]

$$\alpha u_{\mathrm{L}}(\mathbf{k}) = \langle 0|d_{\mathrm{R}} u_{\mathrm{R}} u_{\mathrm{L}}|p(\mathbf{k})\rangle, \quad \beta u_{\mathrm{L}}(\mathbf{k}) = \langle 0|d_{\mathrm{L}} u_{\mathrm{L}} u_{\mathrm{R}}|p(\mathbf{k})\rangle. \quad (57)$$

The $u_{\mathrm{L}}(\mathbf{k})$ denote the left-handed components of the proton wave function. It is known that $|\alpha| = |\beta|$, and β is in the range [26]

$$0.003\,\mathrm{GeV}^3 \le \beta \le 0.03\,\mathrm{GeV}^3. \quad (58)$$

From recent lattice calculations, one group reported that [27]

$$\alpha = -(0.015 \pm 0.001)\,\mathrm{GeV}^3, \quad \beta = 0.014 \pm 0.001\,\mathrm{GeV}^3. \quad (59)$$

But the other group reported the smaller values [28]

$$\alpha = -(0.006 \pm 0.001)\,\mathrm{GeV}^3, \quad \beta = 0.007 \pm 0.001\,\mathrm{GeV}^3. \quad (60)$$

A detailed numerical analysis of the proton decay rate is given in [29].

7 Gauge coupling unification

In general, the gauge coupling unification imposes constraints on the mass spectrum of many varieties of Higgs fields [30]. Our strategy is a generic one in that all of the dimensionless coefficients should remain of order one to preserve the perturbative limit and put all the VEVs at the GUT scale in order to realize the simple gauge coupling unification picture. For the numerical evaluation, we use the one-loop renormalization group equations (RGEs) in the $\overline{\mathrm{DR}}$ scheme [31],[4,5]

$$\frac{1}{\alpha_i(M_G)} = \left.\frac{1}{\alpha_i(M_Z)}\right|_{\overline{\mathrm{MS}}} - \frac{C_2(G_i)}{12\pi} + \frac{1}{2\pi}\left[b_i \log\left(\frac{M_Z}{M_G}\right) + \sum_\zeta b_i^\zeta \log\left(\frac{\det' M_\zeta}{M_G^{\mathrm{rank}(M_\zeta)}}\right)\right], \quad (61)$$

where C_2 is the quadratic Casimir operator; $C_2(\mathrm{SU}(3)) = 3$, $C_2(\mathrm{SU}(2)) = 2$, $C_2(\mathrm{U}(1)) = 0$, and ζ denotes the Higgs fields which have the corresponding gauge quantum numbers. M_ζ is its mass matrix and "det′" means that the determinant should be taken excluding the zero modes. b_i and b_i^ζ are the β function coefficients; $b_3 = -3$, $b_2 = 1$, $b_1 = \frac{33}{5}$, and b_i^ζ are given in Tables 1 and 2. For $\alpha_i(M_Z)|_{\overline{\mathrm{MS}}}$, we use the following values:

$$\alpha_3(M_Z)|_{\overline{\mathrm{MS}}} = \alpha_{\mathrm{s}}(M_Z), \quad (62)$$

$$\alpha_2(M_Z)|_{\overline{\mathrm{MS}}} = \alpha(M_Z)/\sin^2\theta_{\mathrm{W}}(M_Z), \quad (63)$$

$$\alpha_1(M_Z)|_{\overline{\mathrm{MS}}} = \frac{5}{3}\alpha(M_Z)/\left(1 - \sin^2\theta_{\mathrm{W}}(M_Z)\right), \quad (64)$$

with [32]

$$\alpha_{\mathrm{s}}(M_Z) = 0.1172, \quad \alpha(M_Z) = 1/128.92, \quad \sin^2\theta_{\mathrm{W}}(M_Z) = 0.23113. \quad (65)$$

Excluding the fields which mix with the would-be NG fields and the fields with $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_Y$ quantum numbers, $\left[\left(\mathbf{1}, \mathbf{2}, \frac{1}{2}\right) + \mathrm{h.c.}\right]$ and $\left[\left(\overline{\mathbf{3}}, 1, \frac{1}{3}\right) + \mathrm{h.c.}\right]$, the massive fields are given as follows.

For the $\overline{\mathbf{126}}$ and $\mathbf{126}$ representation fields, their quantum numbers, the masses and their β function coefficients are given in Table 1.

For the **210** representation field, their quantum numbers, the masses and their β function coefficients are given in Table 2.

Putting these values into (61), the unification condition produces two individual equations,

$$\alpha_3(M_G) = \alpha_2(M_G), \quad (66)$$

and

$$\alpha_3(M_G) = \alpha_1(M_G). \quad (67)$$

Setting all VEVs at the GUT scale, $\phi_1 \sim \phi_2 \sim \phi_3 \sim |v_{\mathrm{R}}| \sim M_G$, and the remaining dimensionless coefficients of order one; we can search whether (66) and (67) have a solution for M_G below the Planck scale, $M_G \le M_{\mathrm{Planck}}$. If such a solution exists, it would limit the parameters in the superpotential (3) to some restricted region.

8 Conclusion

We find the general formulation for the proton decay rate in the minimal renormalizable SUSY SO(10) models. Using this generic formulation one can find whether the minimal SUSY SO(10) grand unified theory has been excluded.

Recently, using their Yukawa couplings ((8) and (9) in [33]), Goh–Mohapatra–Nasri–Ng obtained the allowed region of (x, y, z) which corresponds to $\left(\frac{a}{d}, -\frac{b}{d}, -\frac{c}{d}\right)$ in our notation. However, they did not discuss the concrete form of the superpotential and, therefore, compatibilities

[4] $\overline{\mathrm{DR}}$ uses dimensional regularization through dimensional reduction with modified minimal subtraction.

[5] Here we assume, for simplicity, that all the mass eigenvalues of the Higgs fields are smaller than M_G and all the masses of the gauge fields lie around M_G. In the other cases, the formula becomes quite complicated.

Table 1. The mass matrices and the β function coefficients for $\overline{\mathbf{126}}$ and $\mathbf{126}$

quantum numbers	mass matrices, or mass eigenvalues	b_3^ζ	b_2^ζ	b_1^ζ
$\left(\mathbf{8},\mathbf{2},\frac{1}{2}\right)+\text{h.c.}$	$\begin{pmatrix} m_2-\frac{\lambda_2\phi_2}{30\sqrt{2}}-\frac{\lambda_2\phi_3}{60} & 0 \\ 0 & m_2-\frac{\lambda_2\phi_2}{30\sqrt{2}}+\frac{\lambda_2\phi_3}{60} \end{pmatrix}$	12	8	$\frac{24}{5}$
$\left(\mathbf{6},\mathbf{3},\frac{1}{3}\right)+\text{h.c.}$	$m_2-\frac{\lambda_2\phi_1}{10\sqrt{6}}-\frac{\lambda_2\phi_2}{30\sqrt{2}}$	15	24	$\frac{12}{5}$
$\left(\mathbf{6},\mathbf{1},\frac{4}{3}\right)+\text{h.c.}$	$m_2+\frac{\lambda_2\phi_1}{10\sqrt{6}}-\frac{\lambda_2\phi_2}{30\sqrt{2}}-\frac{\lambda_2\phi_3}{30}$	5	0	$\frac{64}{5}$
$\left(\overline{\mathbf{6}},\mathbf{1},\frac{2}{3}\right)+\text{h.c.}$	$m_2+\frac{\lambda_2\phi_1}{10\sqrt{6}}-\frac{\lambda_2\phi_2}{30\sqrt{2}}+\frac{\lambda_2\phi_3}{30}$	5	0	$\frac{16}{5}$
$\left(\mathbf{6},\mathbf{1},\frac{1}{3}\right)+\text{h.c.}$	$m_2+\frac{\lambda_2\phi_1}{10\sqrt{6}}-\frac{\lambda_2\phi_2}{30\sqrt{2}}$	5	0	$\frac{4}{5}$
$\left(\overline{\mathbf{3}},\mathbf{3},\frac{1}{3}\right)+\text{h.c.}$	$m_2-\frac{\lambda_2\phi_1}{10\sqrt{6}}+\frac{\lambda_2\phi_2}{30\sqrt{2}}$	3	12	$\frac{6}{5}$
$\left(\mathbf{3},\mathbf{2},\frac{7}{6}\right)+\text{h.c.}$	$\begin{pmatrix} m_2+\frac{\lambda_2\phi_2}{30\sqrt{2}}-\frac{\lambda_2\phi_3}{60} & 0 \\ 0 & m_2+\frac{\lambda_2\phi_2}{30\sqrt{2}}-\frac{\lambda_2\phi_3}{20} \end{pmatrix}$	2	3	$\frac{49}{5}$
$\left(\overline{\mathbf{3}},\mathbf{1},\frac{4}{3}\right)+\text{h.c.}$	$m_2+\frac{\lambda_2\phi_1}{10\sqrt{6}}+\frac{\lambda_2\phi_2}{30\sqrt{2}}-\frac{\lambda_2\phi_3}{30}$	1	0	$\frac{32}{5}$
$(\mathbf{1},\mathbf{3},1)+\text{h.c.}$	$m_2-\frac{\lambda_2\phi_1}{10\sqrt{6}}+\frac{\lambda_2\phi_2}{10\sqrt{2}}$	0	4	$\frac{18}{5}$
$(\mathbf{1},\mathbf{1},2)+\text{h.c.}$	$m_2+\frac{\lambda_2\phi_1}{10\sqrt{6}}+\frac{\lambda_2\phi_2}{10\sqrt{2}}-\frac{\lambda_2\phi_3}{10}$	0	0	$\frac{24}{5}$

Table 2. The mass matrices and the β function coefficients for $\mathbf{210}$

quantum numbers	mass matrices, or mass eigenvalues	b_3^ζ	b_2^ζ	b_1^ζ
$(\mathbf{8},\mathbf{3},0)$	$2m_1-\frac{\lambda_1\phi_1}{\sqrt{6}}-\frac{\lambda_1\phi_2}{3\sqrt{2}}$	9	16	0
$(\mathbf{8},\mathbf{1},1)+\text{h.c.}$	$2m_1+\frac{\lambda_1\phi_1}{\sqrt{6}}-\frac{\lambda_1\phi_2}{3\sqrt{2}}$	6	0	$\frac{48}{5}$
$(\mathbf{8},\mathbf{1},0)$	$\begin{pmatrix} 2m_1-\frac{\lambda_1\phi_2}{3\sqrt{2}} & \frac{\lambda_1\phi_3}{3\sqrt{2}} \\ \frac{\lambda_1\phi_3}{3\sqrt{2}} & 2m_1+\frac{\lambda_1\phi_1}{\sqrt{6}}-\frac{\lambda_1\phi_2}{3\sqrt{2}} \end{pmatrix}$	3	0	0
$\left(\mathbf{6},\mathbf{2},\frac{5}{6}\right)+\text{h.c.}$	$2m_1-\frac{\lambda_1\phi_2}{3\sqrt{2}}-\frac{\lambda_1\phi_3}{6}$	10	6	10
$\left(\overline{\mathbf{6}},\mathbf{2},\frac{1}{6}\right)+\text{h.c.}$	$2m_1-\frac{\lambda_1\phi_2}{3\sqrt{2}}+\frac{\lambda_1\phi_3}{6}$	10	6	$\frac{2}{5}$
$\left(\mathbf{3},\mathbf{3},\frac{2}{3}\right)+\text{h.c.}$	$2m_1-\frac{\lambda_1\phi_1}{\sqrt{6}}+\frac{\lambda_1\phi_2}{3\sqrt{2}}$	3	12	$\frac{24}{5}$
$\left(\mathbf{3},\mathbf{1},\frac{5}{3}\right)+\text{h.c.}$	$2m_1+\frac{\lambda_1\phi_1}{\sqrt{6}}+\frac{\lambda_1\phi_2}{3\sqrt{2}}-\frac{2\lambda_1\phi_3}{3}$	1	0	10
$(\mathbf{1},\mathbf{3},0)$	$2m_1-\frac{\lambda_1\phi_1}{\sqrt{6}}+\frac{\sqrt{2}\lambda_1\phi_2}{3}$	0	2	0
$\left(\mathbf{1},\mathbf{2},\frac{3}{2}\right)+\text{h.c.}$	$2m_1+\frac{\lambda_1\phi_2}{\sqrt{2}}-\frac{\lambda_1\phi_3}{2}$	0	1	$\frac{27}{5}$

of their superpotential with the other constraints are not clear in their paper. Also, as we have mentioned above, there appears a non-zero x value even without the **54** dimensional Higgs field. Further, besides the color triplet Higgs fields, there is a much richer Higgs particle contents. These additional Higgs fields may cause a pathology of the gauge coupling unification. This paper presents a relationship among these comprehensive but tightly connected problems.

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Appendix A: Dimension-five operators

In this appendix, we list the explicit form of the various interaction coefficients.

We use the following notation for the mixing matrices which diagonalize the squark, slepton mass-squared matrices and chargino, neutralino mass matrices. Squark, slepton mass-squared matrix $M^2_{\tilde f}$, chargino and neutralino mass matrices M_C and M_N are diagonalized by the unitary matrices $U_{\tilde f}$, O_{L}, O_{R} and O_N, respectively. We have

$$\begin{aligned} &{}_{\tilde f}\, M^2_{\tilde f}\, U^\dagger_{\tilde f}\mathrm{diag}(m^2_{\tilde f_1},....,m^2_{\tilde f_6}), \\ &O_{\mathrm{R}}\, M_C\, O^\dagger_{\mathrm{L}}\mathrm{diag}(m_{\tilde\chi_1^-},m_{\tilde\chi_2^-}), \\ &O^*_N\, M_N\, O^\dagger_N\mathrm{diag}(m_{\tilde\chi_1^0},m_{\tilde\chi_2^0},m_{\tilde\chi_3^0},m_{\tilde\chi_4^0}). \end{aligned} \quad \text{(A.1)}$$

For the dimension-five operators, we have the following expressions:[6]

$$C_{\mathrm{L}}^{(\tilde u\tilde d u e)XYij} \equiv C_{\mathrm{L}}^{[ijk]l}(U^*_{\tilde u})_{Xk}(U^*_{\tilde d})_{Yl}, \quad \text{(A.2)}$$

[6] We use a notation for an antisymmetric tensor, $A^{[ijk]l} \equiv A^{ijkl} - A^{kjil}$.

$$C_{\mathrm{L}}^{(\tilde{u}\tilde{u}de)XYij} \equiv C_{\mathrm{L}}^{[kjl]m}(U_{\tilde{u}}^*)_{Xk}(U_{\tilde{u}}^*)_{Yl}(V_{\mathrm{CKM}})_{im}, \quad \text{(A.3)}$$

$$C_{\mathrm{R}}^{(\tilde{u}\tilde{d}ue)XYij} \equiv (C_{\mathrm{R}}^{*klji} - C_{\mathrm{R}}^{*iljk})(U_{\tilde{u}}^*)_{X,k+3}(U_{\tilde{d}}^*)_{Y,l+3}, \quad \text{(A.4)}$$

$$C_{\mathrm{R}}^{(\tilde{u}\tilde{u}de)XYij} \equiv (C_{\mathrm{R}}^{*klji} - C_{\mathrm{R}}^{*iljk})(U_{\tilde{u}}^*)_{X,k+3}(U_{\tilde{u}}^*)_{Y,l+3}, \quad \text{(A.5)}$$

$$C_{\mathrm{L}}^{(\tilde{u}\tilde{d}d\nu)XYij} \equiv (C_{\mathrm{L}}^{mnkl} - C_{\mathrm{L}}^{lknm})(U_{\tilde{u}}^*)_{Xk}(U_{\tilde{d}}^*)_{Yl} \times (V_{\mathrm{CKM}})_{im}(U_{\mathrm{MNS}})_{jn}, \quad \text{(A.6)}$$

$$C_{\mathrm{L}}^{(\tilde{d}\tilde{d}u\nu)XYij} \equiv (C_{\mathrm{L}}^{lnik} - C_{\mathrm{L}}^{knil})(U_{\tilde{d}}^*)_{Xk}(U_{\tilde{d}}^*)_{Yl}(U_{\mathrm{MNS}})_{jn}, \quad \text{(A.7)}$$

$$C_{\mathrm{L}}^{(\tilde{u}\tilde{e}ud)XYij} \equiv C_{\mathrm{L}}^{[kli]m}(U_{\tilde{u}}^*)_{Xk}(U_{\tilde{e}}^*)_{Yl}(V_{\mathrm{CKM}})_{jm}, \quad \text{(A.8)}$$

$$C_{\mathrm{L}}^{(\tilde{d}\tilde{e}uu)XYij} \equiv C_{\mathrm{L}}^{[ilj]k}(U_{\tilde{d}}^*)_{Xk}(U_{\tilde{e}}^*)_{Yl}, \quad \text{(A.9)}$$

$$C_{\mathrm{R}}^{(\tilde{u}\tilde{e}ud)XYij} \equiv (C_{\mathrm{R}}^{*jkli} - C_{\mathrm{R}}^{*kjli})(U_{\tilde{u}}^*)_{X,k+3}(U_{\tilde{e}}^*)_{Y,l+3}, \quad \text{(A.10)}$$

$$C_{\mathrm{R}}^{(\tilde{d}\tilde{e}uu)XYij} \equiv (C_{\mathrm{R}}^{*jkli} - C_{\mathrm{R}}^{*iklj})(U_{\tilde{d}}^*)_{X,k+3}(U_{\tilde{e}}^*)_{Y,l+3}, \quad \text{(A.11)}$$

$$C_{\mathrm{L}}^{(\tilde{d}\tilde{\nu}ud)XYij} \equiv (C_{\mathrm{L}}^{klim} - C_{\mathrm{L}}^{mlik})(U_{\tilde{d}}^*)_{Xk}(U_{\tilde{\nu}}^*)_{Yl}(V_{\mathrm{CKM}})_{jm}, \quad \text{(A.12)}$$

$$C_{\mathrm{L}}^{(\tilde{u}\tilde{\nu}dd)XYij} \equiv (C_{\mathrm{L}}^{nlkm} - C_{\mathrm{L}}^{mlkn})(U_{\tilde{u}}^*)_{Xk}(U_{\tilde{\nu}}^*)_{Yl}(V_{\mathrm{CKM}})_{im} \times (V_{\mathrm{CKM}})_{jn}. \quad \text{(A.13)}$$

In (A.6) and (A.7), it should be noticed that the neutrinos in the final states should be rotated from the flavor eigenstates to the mass eigenstates by using the Maki–Nakagawa–Sakata (MNS) mixing matrix [36], U_{MNS}.

Appendix B: Sparticles interactions

We use the following notation for the quark–gluino–squark, quark (lepton)–chargino–squark (slepton) and quark (lepton)–neutralino–squark (slepton) interactions.
(1) quark–gluino–squark interactions:

$$\mathcal{L}_{int} = -\mathrm{i}\sqrt{2}u_i^c\left[G_{iX}^{\mathrm{L}(u)}P_{\mathrm{L}} + G_{iX}^{\mathrm{R}(u)}P_{\mathrm{R}}\right]\widetilde{g}\widetilde{u}_X - \mathrm{i}\sqrt{2}d_i^c\left[G_{iX}^{\mathrm{L}(d)}P_{\mathrm{L}} + G_{iX}^{\mathrm{R}(d)}P_{\mathrm{R}}\right]\widetilde{g}\widetilde{d}_X + \mathrm{h.c.} \quad \text{(B.1)}$$

(2) quark (lepton)–chargino–squark (slepton) interactions:

$$\mathcal{L}_{int} = u_i^c\left[C_{iAX}^{\mathrm{L}(u)}P_{\mathrm{L}} + C_{iAX}^{\mathrm{R}(u)}P_{\mathrm{R}}\right]\widetilde{\chi}_A^+\widetilde{d}_X + d_i^c\left[C_{iAX}^{\mathrm{L}(d)}P_{\mathrm{L}} + C_{iAX}^{\mathrm{R}(d)}P_{\mathrm{R}}\right]\widetilde{\chi}_A^+\widetilde{u}_X + \nu_i^c C_{iAX}^{\mathrm{R}(\nu)}P_{\mathrm{R}}\widetilde{\chi}_A^+\widetilde{e}_X + e_i^c\left[C_{iAX}^{\mathrm{L}(e)}P_{\mathrm{L}} + C_{iAX}^{\mathrm{R}(e)}P_{\mathrm{R}}\right]\widetilde{\chi}_A^+\widetilde{\nu}_X + \mathrm{h.c.} \quad \text{(B.2)}$$

(3) quark (lepton)–neutralino–squark (slepton) interactions:

$$\mathcal{L}_{int} = u_i^c\left[N_{iAX}^{\mathrm{L}(u)}P_{\mathrm{L}} + N_{iAX}^{\mathrm{R}(u)}P_{\mathrm{R}}\right]\widetilde{\chi}_A^0\widetilde{u}_X + d_i^c\left[N_{iAX}^{\mathrm{L}(d)}P_{\mathrm{L}} + N_{iAX}^{\mathrm{R}(d)}P_{\mathrm{R}}\right]\widetilde{\chi}_A^0\widetilde{d}_X + \nu_i^c N_{iAX}^{\mathrm{R}(\nu)}P_{\mathrm{R}}\widetilde{\chi}_A^0\widetilde{\nu}_X + e_i^c\left[N_{iAX}^{\mathrm{L}(e)}P_{\mathrm{L}} + N_{iAX}^{\mathrm{R}(e)}P_{\mathrm{R}}\right]\widetilde{\chi}_A^0\widetilde{e}_X + \mathrm{h.c.} \quad \text{(B.3)}$$

Explicitly, we have the following expressions:

$$G_{iX}^{\mathrm{L}(u)} \equiv g_3(U_{\tilde{u}}^*)_{X,i+3}, \quad \text{(B.4)}$$

$$G_{iX}^{\mathrm{R}(u)} \equiv g_3(U_{\tilde{u}}^*)_{Xi}, \quad \text{(B.5)}$$

$$G_{iX}^{\mathrm{L}(d)} \equiv g_3(U_{\tilde{d}}^*)_{X,i+3}, \quad \text{(B.6)}$$

$$G_{iX}^{\mathrm{R}(d)} \equiv g_3(U_{\tilde{d}}^*)_{Xk}(V_{\mathrm{CKM}}^*)_{ik}, \quad \text{(B.7)}$$

$$C_{iAX}^{\mathrm{L}(u)} \equiv g\frac{m_{u_i}}{\sqrt{2}M_W\sin\beta}(O_{\mathrm{R}}^*)_{A2}(U_{\tilde{d}}^*)_{Xi}, \quad \text{(B.8)}$$

$$C_{iAX}^{\mathrm{R}(u)} \equiv g\left\{-(O_{\mathrm{L}}^*)_{A1}(U_{\tilde{d}}^*)_{Xi} + \frac{m_{d_i}}{\sqrt{2}M_W\cos\beta}(O_{\mathrm{L}}^*)_{A2}(U_{\tilde{u}}^*)_{X,i+3}\right\}, \quad \text{(B.9)}$$

$$C_{iAX}^{\mathrm{L}(d)} \equiv g\frac{m_{d_i}}{\sqrt{2}M_W\cos\beta}(O_{\mathrm{L}}^*)_{A2}(U_{\tilde{u}}^*)_{Xi}, \quad \text{(B.10)}$$

$$C_{iAX}^{\mathrm{R}(d)} \equiv g\left\{-(O_{\mathrm{R}}^*)_{A1}(U_{\tilde{u}}^*)_{Xk} + \frac{m_{u_k}}{\sqrt{2}M_W\sin\beta}(O_{\mathrm{R}}^*)_{A2}(U_{\tilde{u}}^*)_{X,k+3}\right\}(V_{\mathrm{CKM}}^*)_{ik}, \quad \text{(B.11)}$$

$$C_{iAX}^{\mathrm{R}(\nu)} \equiv g\left\{-(O_{\mathrm{L}}^*)_{A1}(U_{\tilde{d}}^*)_{Xk} + \frac{m_{e_k}}{\sqrt{2}M_W\cos\beta}(O_{\mathrm{L}}^*)_{A2}(U_{\tilde{\nu}}^*)_{X,k+3}\right\}(U_{\mathrm{MNS}}^*)_{ik}, \quad \text{(B.12)}$$

$$C_{iAX}^{\mathrm{L}(e)} \equiv g\frac{m_{e_i}}{\sqrt{2}M_W\cos\beta}(O_{\mathrm{L}}^*)_{A2}(U_{\tilde{\nu}}^*)_{Xi}, \quad \text{(B.13)}$$

$$C_{iAX}^{\mathrm{R}(e)} \equiv -g\left\{-(O_{\mathrm{R}}^*)_{A1}(U_{\tilde{\nu}}^*)_{Xk} + \frac{m_{u_k}}{\sqrt{2}M_W\sin\beta}(O_{\mathrm{R}}^*)_{A2}(U_{\tilde{u}}^*)_{X,k+3}\right\}(V_{\mathrm{CKM}}^*)_{ik}, \quad \text{(B.14)}$$

$$N_{iAX}^{\mathrm{L}(u)} \equiv -\frac{g}{\sqrt{2}}\left\{\frac{m_{u_i}}{M_W\sin\beta}(O_N^*)_{A4}(U_{\tilde{u}}^*)_{Xi} - \frac{4}{3}\tan\theta_{\mathrm{W}}(O_N^*)_{A1}(U_{\tilde{u}}^*)_{X,i+3}\right\}, \quad \text{(B.15)}$$

$$N_{iAX}^{\mathrm{R}(u)} \equiv -\frac{g}{\sqrt{2}}\left\{\frac{m_{u_i}}{M_W\sin\beta}(O_N^*)_{A4}(U_{\tilde{u}}^*)_{X,i+3} + \left[(O_N^*)_{A2} + \frac{1}{3}\tan\theta_{\mathrm{W}}(O_N^*)_{A1}\right](U_{\tilde{u}}^*)_{Xi}\right\}, \quad \text{(B.16)}$$

$$N_{iAX}^{\mathrm{L}(d)} \equiv -\frac{g}{\sqrt{2}}\left\{\frac{m_{d_i}}{M_W\cos\beta}(O_N^*)_{A3}(U_{\tilde{d}}^*)_{Xi} + \frac{2}{3}\tan\theta_{\mathrm{W}}(O_N^*)_{A1}(U_{\tilde{d}}^*)_{X,i+3}\right\}, \quad \text{(B.17)}$$

$$N^{\mathrm{R}(d)}_{iAX} \equiv -\frac{g}{\sqrt{2}}\left\{\frac{m_{d_k}}{M_W\cos\beta}(O^*_N)_{A3}(U^*_{\tilde{d}})_{X,k+3} + \left[-(O^*_N)_{A2}+\frac{1}{3}\tan\theta_{\mathrm{W}}(O^*_N)_{A1}\right](U^*_{\tilde{d}})_{Xk}\right\} \times (V^*_{\mathrm{CKM}})_{ik}, \tag{B.18}$$

$$N^{\mathrm{R}(\nu)}_{iAX} \equiv -\frac{g}{\sqrt{2}}\left[(O^*_N)_{A2}-\tan\theta_{\mathrm{W}}(O^*_N)_{A1}\right] \times (U^*_{\tilde{\nu}})_{X,k}(U^*_{\mathrm{MNS}})_{ik}, \tag{B.19}$$

$$N^{\mathrm{L}(e)}_{iAX} \equiv -\frac{g}{\sqrt{2}}\left\{\frac{m_{e_i}}{M_W\cos\beta}(O^*_N)_{A3}(U^*_{\tilde{e}})_{Xi} + \frac{2}{3}\tan\theta_{\mathrm{W}}(O^*_N)_{A1}(U^*_{\tilde{e}})_{X,i+3}\right\}, \tag{B.20}$$

$$N^{\mathrm{R}(e)}_{iAX} \equiv -\frac{g}{\sqrt{2}}\left\{\frac{m_{e_i}}{M_W\cos\beta}(O^*_N)_{A3}(U^*_{\tilde{e}})_{X,i+3} + \left[-(O^*_N)_{A2}+\frac{1}{3}\tan\theta_{\mathrm{W}}(O^*_N)_{A1}\right](U^*_{\tilde{e}})_{Xi}\right\}. \tag{B.21}$$

These expressions are found in [37], but only for the quark sector. So here we write them explicitly.

Appendix C: Dimension-six operators

For the dimension-six operator, we divide the coefficients into three parts according to the dressed sparticles,

$$C^{(udue)ij}_{\mathrm{LL}} = C^{(udue)ij}_{\mathrm{LL}}(\tilde{g}) + C^{(udue)ij}_{\mathrm{LL}}(\tilde{\chi}^0) + C^{(udue)ij}_{\mathrm{LL}}(\tilde{\chi}^\pm), \tag{C.1}$$

etc. Then we have the following expressions. These expressions have the same forms as [25]. However, ours are different from them in the neutrino sector as was mentioned in the end of Appendix A. We have

$$C^{(udue)ij}_{\mathrm{LL}}(\tilde{g}) \tag{C.2}$$
$$= \frac{4}{3}\frac{1}{m_{\tilde{g}}}, C^{(udue)XY1j}_{\mathrm{L}} G^{\mathrm{R}(u)}_{1X} G^{\mathrm{R}(d)}_{iY} F\left(\frac{m^2_{\tilde{g}}}{m^2_{\tilde{u}_X}}, \frac{m^2_{\tilde{g}}}{m^2_{\tilde{d}_Y}}\right),$$

$$C^{(udue)ij}_{\mathrm{LL}}(\tilde{\chi}^\pm)$$
$$= \frac{1}{m_{\tilde{\chi}^+_A}}\left[-C^{(udue)XY1j}_{\mathrm{L}} C^{\mathrm{R}(u)}_{1AY} C^{\mathrm{R}(d)}_{iAX} F\left(\frac{m^2_{\tilde{\chi}^+_A}}{m^2_{\tilde{u}_X}}, \frac{m^2_{\tilde{\chi}^+_A}}{m^2_{\tilde{d}_Y}}\right) + C^{(d\nu ud)XY1i}_{\mathrm{L}} C^{\mathrm{R}(d)}_{1AX} C^{\mathrm{R}(e)}_{jAY} F\left(\frac{m^2_{\tilde{\chi}^+_A}}{m^2_{\tilde{d}_X}}, \frac{m^2_{\tilde{\chi}^+_A}}{m^2_{\tilde{\nu}_Y}}\right)\right], \tag{C.3}$$

$$C^{(udue)ij}_{\mathrm{LL}}(\tilde{\chi}^0)$$
$$= \frac{1}{m_{\tilde{\chi}^0_A}}\left[C^{(udue)XY1j}_{\mathrm{L}} N^{\mathrm{R}(u)}_{1AX} N^{\mathrm{R}(d)}_{iAY} F\left(\frac{m^2_{\tilde{\chi}^0_A}}{m^2_{\tilde{u}_X}}, \frac{m^2_{\tilde{\chi}^0_A}}{m^2_{\tilde{d}_Y}}\right) + C^{(ueud)XY1i}_{\mathrm{L}} N^{\mathrm{R}(d)}_{1AX} N^{\mathrm{R}(e)}_{jAY} F\left(\frac{m^2_{\tilde{\chi}^0_A}}{m^2_{\tilde{d}_X}}, \frac{m^2_{\tilde{\chi}^0_A}}{m^2_{\tilde{\nu}_Y}}\right)\right], \tag{C.4}$$

$$C^{(udue)ij}_{\mathrm{RL}}(\tilde{g})$$
$$= \frac{4}{3}\frac{1}{m_{\tilde{g}}} C^{(udue)XY1j}_{\mathrm{L}} G^{\mathrm{L}(u)}_{1X} G^{\mathrm{L}(d)}_{iY} F\left(\frac{m^2_{\tilde{g}}}{m^2_{\tilde{u}_X}}, \frac{m^2_{\tilde{g}}}{m^2_{\tilde{d}_Y}}\right), \tag{C.5}$$

$$C^{(udue)ij}_{\mathrm{RL}}(\tilde{\chi}^\pm) \tag{C.6}$$
$$= -\frac{1}{m_{\tilde{\chi}^+_A}} C^{(udue)XY1j}_{\mathrm{L}} C^{\mathrm{L}(u)}_{1AY} C^{\mathrm{L}(d)}_{iAX} F\left(\frac{m^2_{\tilde{\chi}^+_A}}{m^2_{\tilde{u}_X}}, \frac{m^2_{\tilde{\chi}^+_A}}{m^2_{\tilde{d}_Y}}\right),$$

$$C^{(udue)ij}_{\mathrm{RL}}(\tilde{\chi}^0)$$
$$= \frac{1}{m_{\tilde{\chi}^0_A}}\left[C^{(udue)XY1j}_{\mathrm{L}} N^{\mathrm{L}(u)}_{1AX} N^{\mathrm{L}(d)}_{iAY} F\left(\frac{m^2_{\tilde{\chi}^0_A}}{m^2_{\tilde{u}_X}}, \frac{m^2_{\tilde{\chi}^0_A}}{m^2_{\tilde{d}_Y}}\right) + C^{(ueud)XY1i}_{\mathrm{R}} N^{\mathrm{R}(d)}_{1AX} N^{\mathrm{R}(e)}_{jAY} F\left(\frac{m^2_{\tilde{\chi}^0_A}}{m^2_{\tilde{u}_X}}, \frac{m^2_{\tilde{\chi}^0_A}}{m^2_{\tilde{e}_Y}}\right)\right], \tag{C.7}$$

$$C^{(udue)ij}_{\mathrm{LR}}(\tilde{g})$$
$$= \frac{4}{3}\frac{1}{m_{\tilde{g}}} C^{(udue)XY1j}_{\mathrm{R}} G^{\mathrm{R}(u)}_{1X} G^{\mathrm{R}(d)}_{iY} F\left(\frac{m^2_{\tilde{g}}}{m^2_{\tilde{u}_X}}, \frac{m^2_{\tilde{g}}}{m^2_{\tilde{d}_Y}}\right), \tag{C.8}$$

$$C^{(udue)ij}_{\mathrm{LR}}(\tilde{\chi}^\pm)$$
$$= \frac{1}{m_{\tilde{\chi}^+_A}}\left[-C^{(udue)XY1j}_{\mathrm{R}} C^{\mathrm{R}(u)}_{1AY} C^{\mathrm{R}(d)}_{iAX} F\left(\frac{m^2_{\tilde{\chi}^+_A}}{m^2_{\tilde{u}_X}}, \frac{m^2_{\tilde{\chi}^+_A}}{m^2_{\tilde{d}_Y}}\right) + C^{(d\nu ud)XY1i}_{\mathrm{L}} C^{\mathrm{L}(d)}_{1AX} C^{\mathrm{L}(e)}_{jAY} F\left(\frac{m^2_{\tilde{\chi}^+_A}}{m^2_{\tilde{d}_X}}, \frac{m^2_{\tilde{\chi}^+_A}}{m^2_{\tilde{\nu}_Y}}\right)\right], \tag{C.9}$$

$$C^{(udue)ij}_{\mathrm{LR}}(\tilde{\chi}^0)$$
$$= \frac{1}{m_{\tilde{\chi}^0_A}}\left[C^{(udue)XY1j}_{\mathrm{R}} N^{\mathrm{R}(u)}_{1AX} N^{\mathrm{R}(d)}_{iAY} F\left(\frac{m^2_{\tilde{\chi}^0_A}}{m^2_{\tilde{u}_X}}, \frac{m^2_{\tilde{\chi}^0_A}}{m^2_{\tilde{d}_Y}}\right) + C^{(ueud)XY1i}_{\mathrm{L}} N^{\mathrm{L}(d)}_{1AX} N^{\mathrm{L}(e)}_{jAY} F\left(\frac{m^2_{\tilde{\chi}^0_A}}{m^2_{\tilde{d}_X}}, \frac{m^2_{\tilde{\chi}^0_A}}{m^2_{\tilde{e}_Y}}\right)\right], \tag{C.10}$$

$$C^{(udue)ij}_{\mathrm{RR}}(\tilde{g}) \tag{C.11}$$
$$= \frac{4}{3}\frac{1}{m_{\tilde{g}}} C^{(udue)XY1j}_{\mathrm{R}} G^{\mathrm{L}(u)}_{1X} G^{\mathrm{L}(d)}_{iY} F\left(\frac{m^2_{\tilde{g}}}{m^2_{\tilde{u}_X}}, \frac{m^2_{\tilde{g}}}{m^2_{\tilde{d}_Y}}\right),$$

$$C^{(udue)ij}_{\mathrm{RR}}(\tilde{\chi}^\pm) \tag{C.12}$$
$$= -\frac{1}{m_{\tilde{\chi}^+_A}} C^{(udue)XY1j}_{\mathrm{R}} C^{\mathrm{L}(u)}_{1AY} C^{\mathrm{L}(d)}_{iAX} F\left(\frac{m^2_{\tilde{\chi}^+_A}}{m^2_{\tilde{u}_X}}, \frac{m^2_{\tilde{\chi}^+_A}}{m^2_{\tilde{d}_Y}}\right),$$

$$C^{(udue)ij}_{\mathrm{RR}}(\tilde{\chi}^0)$$
$$= \frac{1}{m_{\tilde{\chi}^0_A}}\left[C^{(udue)XY1j}_{\mathrm{L}} N^{\mathrm{L}(u)}_{1AX} N^{\mathrm{L}(d)}_{iAY} F\left(\frac{m^2_{\tilde{\chi}^0_A}}{m^2_{\tilde{u}_X}}, \frac{m^2_{\tilde{\chi}^0_A}}{m^2_{\tilde{d}_Y}}\right) + C^{(ueud)XY1i}_{\mathrm{R}} N^{\mathrm{R}(d)}_{1AX} N^{\mathrm{R}(e)}_{jAY} F\left(\frac{m^2_{\tilde{\chi}^0_A}}{m^2_{\tilde{u}_X}}, \frac{m^2_{\tilde{\chi}^0_A}}{m^2_{\tilde{e}_Y}}\right)\right], \tag{C.13}$$

$$C^{(udd\nu)ijk}_{\mathrm{LL}}(\tilde{g})$$
$$= \frac{4}{3}\frac{1}{m_{\tilde{g}}}\left[C^{(udd\nu)XYjk}_{\mathrm{L}} G^{\mathrm{R}(u)}_{1X} G^{\mathrm{R}(d)}_{iY} F\left(\frac{m^2_{\tilde{g}}}{m^2_{\tilde{u}_X}}, \frac{m^2_{\tilde{g}}}{m^2_{\tilde{d}_Y}}\right)\right.$$

$$+ C_{\mathrm{L}}^{(ddu\nu)XY1k} G_{jX}^{\mathrm{R}(d)} G_{iY}^{\mathrm{R}(d)} F\left(\frac{m_{\widetilde{g}}^2}{m_{\widetilde{d}_X}^2}, \frac{m_{\widetilde{g}}^2}{m_{\widetilde{d}_Y}^2}\right)\Bigg], \quad \text{(C.14)}$$

$$C_{\mathrm{LL}}^{(udd\nu)ijk}(\widetilde{\chi}^{\pm})$$
$$= \frac{1}{m_{\widetilde{\chi}_A^+}}\Bigg[-C_{\mathrm{L}}^{(udd\nu)XYjk} C_{1AY}^{\mathrm{R}(u)} C_{iAX}^{\mathrm{R}(d)} F\left(\frac{m_{\widetilde{\chi}_A^+}^2}{m_{\widetilde{u}_X}^2}, \frac{m_{\widetilde{\chi}_A^+}^2}{m_{\widetilde{d}_Y}^2}\right)$$
$$+ C_{\mathrm{L}}^{(ueud)XY1i} C_{jAX}^{\mathrm{R}(u)} C_{kAY}^{\mathrm{R}(e)} F\left(\frac{m_{\widetilde{\chi}_A^+}^2}{m_{\widetilde{d}_X}^2}, \frac{m_{\widetilde{\chi}_A^+}^2}{m_{\widetilde{\nu}_Y}^2}\right)\Bigg], \quad \text{(C.15)}$$

$$C_{\mathrm{LL}}^{(udd\nu)ijk}(\widetilde{\chi}^{0})$$
$$= \frac{1}{m_{\widetilde{\chi}_A^0}}\Bigg[C_{\mathrm{L}}^{(udd\nu)XYjk} N_{1AX}^{\mathrm{R}(u)} N_{iAY}^{\mathrm{R}(d)} F\left(\frac{m_{\widetilde{\chi}_A^0}^2}{m_{\widetilde{u}_X}^2}, \frac{m_{\widetilde{\chi}_A^0}^2}{m_{\widetilde{d}_Y}^2}\right)$$
$$+ C_{\mathrm{L}}^{(ddu\nu)XY1k} N_{jAX}^{\mathrm{R}(d)} N_{iAY}^{\mathrm{R}(e)} F\left(\frac{m_{\widetilde{\chi}_A^0}^2}{m_{\widetilde{d}_X}^2}, \frac{m_{\widetilde{\chi}_A^0}^2}{m_{\widetilde{d}_Y}^2}\right)$$
$$+ C_{\mathrm{L}}^{(d\nu ud)XY1i} N_{jAX}^{\mathrm{R}(d)} N_{kAY}^{\mathrm{R}(e)} F\left(\frac{m_{\widetilde{\chi}_A^0}^2}{m_{\widetilde{d}_X}^2}, \frac{m_{\widetilde{\chi}_A^0}^2}{m_{\widetilde{\nu}_Y}^2}\right)$$
$$+ C_{\mathrm{L}}^{(u\nu dd)XYji} N_{1AX}^{\mathrm{R}(u)} N_{kAY}^{\mathrm{R}(\nu)} F\left(\frac{m_{\widetilde{\chi}_A^0}^2}{m_{\widetilde{u}_X}^2}, \frac{m_{\widetilde{\chi}_A^0}^2}{m_{\widetilde{\nu}_Y}^2}\right)\Bigg], \quad \text{(C.16)}$$

$$C_{\mathrm{RL}}^{(udd\nu)ijk}(\widetilde{g}) \quad \text{(C.17)}$$
$$= \frac{4}{3}\frac{1}{m_{\widetilde{g}}} C_{\mathrm{L}}^{(udd\nu)XYjk} G_{1X}^{\mathrm{L}(u)} G_{iY}^{\mathrm{L}(d)} F\left(\frac{m_{\widetilde{g}}^2}{m_{\widetilde{u}_X}^2}, \frac{m_{\widetilde{g}}^2}{m_{\widetilde{d}_Y}^2}\right),$$

$$C_{\mathrm{RL}}^{(udd\nu)ijk}(\widetilde{\chi}^{\pm})$$
$$= \frac{1}{m_{\widetilde{\chi}_A^+}}\Bigg[-C_{\mathrm{L}}^{(udd\nu)XYjk} C_{1AY}^{\mathrm{L}(u)} C_{iAX}^{\mathrm{L}(d)} F\left(\frac{m_{\widetilde{\chi}_A^+}^2}{m_{\widetilde{u}_X}^2}, \frac{m_{\widetilde{\chi}_A^+}^2}{m_{\widetilde{d}_Y}^2}\right)$$
$$+ C_{\mathrm{R}}^{(ueud)XY1i} C_{jAX}^{\mathrm{R}(u)} C_{kAY}^{\mathrm{R}(e)} F\left(\frac{m_{\widetilde{\chi}_A^+}^2}{m_{\widetilde{d}_X}^2}, \frac{m_{\widetilde{\chi}_A^+}^2}{m_{\widetilde{\nu}_Y}^2}\right)\Bigg], \quad \text{(C.18)}$$

$$C_{\mathrm{RL}}^{(udd\nu)ijk}(\widetilde{\chi}^{0}) \quad \text{(C.19)}$$
$$= \frac{1}{m_{\widetilde{\chi}_A^0}} C_{\mathrm{L}}^{(udd\nu)XYjk} N_{1AX}^{\mathrm{L}(u)} N_{iAY}^{\mathrm{L}(d)} F\left(\frac{m_{\widetilde{\chi}_A^0}^2}{m_{\widetilde{u}_X}^2}, \frac{m_{\widetilde{\chi}_A^0}^2}{m_{\widetilde{d}_Y}^2}\right),$$

$$C_{\mathrm{RL}}^{(ddu\nu)ijk}(\widetilde{g}) \quad \text{(C.20)}$$
$$= \frac{4}{3}\frac{1}{m_{\widetilde{g}}} C_{\mathrm{L}}^{(udd\nu)XY1k} G_{iX}^{\mathrm{L}(d)} G_{jY}^{\mathrm{L}(d)} F\left(\frac{m_{\widetilde{g}}^2}{m_{\widetilde{d}_X}^2}, \frac{m_{\widetilde{g}}^2}{m_{\widetilde{d}_Y}^2}\right),$$

$$C_{\mathrm{RL}}^{(ddu\nu)ijk}(\widetilde{\chi}^{\pm}) = 0, \quad \text{(C.21)}$$

$$C_{\mathrm{RL}}^{(ddu\nu)ijk}(\widetilde{\chi}^{0}) \quad \text{(C.22)}$$
$$= \frac{1}{m_{\widetilde{\chi}_A^0}} C_{\mathrm{L}}^{(ddu\nu)XY1k} N_{iAX}^{\mathrm{L}(d)} N_{jAY}^{\mathrm{L}(d)} F\left(\frac{m_{\widetilde{\chi}_A^0}^2}{m_{\widetilde{d}_X}^2}, \frac{m_{\widetilde{\chi}_A^0}^2}{m_{\widetilde{d}_Y}^2}\right).$$

Here we have defined the loop function

$$F(x,y) \equiv \frac{x\,y}{x-y}\left(\frac{1}{1-x}\log x - \frac{1}{1-y}\log y\right). \quad \text{(C.23)}$$

References

1. M. Shiozawa, talk at Noon 2003 [http://www-sk.icrr.u-tokyo.ac.jp/noon2003/]
2. H. Murayama, A. Pierce, Phys. Rev. D **65**, 055009 (2002)
3. B. Bajc, P.F. Perez, G. Senjanović, Phys. Rev. D **66**, 075005 (2002)
4. T. Yanagida, in Proceedings of the workshop on the Unified Theory and Baryon Number in the Universe, edited by O. Sawada, A. Sugamoto (KEK, Tsukuba 1979); M. Gell-Mann, P. Ramond, R. Slansky, in Supergravity, edited by D. Freedman, P. van Nieuwenhuizen (North-Holland, Amsterdam 1979); R.N. Mohapatra, G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980)
5. K.S. Babu, R.N. Mohapatra, Phys. Rev. Lett. **70**, 2845 (1993)
6. T. Fukuyama, N. Okada, JHEP **0211**, 011 (2002); K. Matsuda, Y. Koide, T. Fukuyama, H. Nishiura, Phys. Rev. D **65**, 033008 (2002) [Erratum D **65**, 079904 (2002)]; K. Matsuda, Y. Koide, T. Fukuyama, Phys. Rev. D **64**, 053015 (2001)
7. S.M. Barr, S. Raby, Phys. Rev. Lett. **79**, 4748 (1997)
8. C.H. Albright, S.M. Barr, Phys. Lett. B **461**, 218 (1999); Phys. Rev. D **62**, 093008 (2000)
9. K. Eguchi et al. [KamLAND Collaboration], Phys. Rev. Lett. **90**, 021802 (2003)
10. S.N. Ahmed et al. [SNO Collaboration], nucl-ex/0309004
11. Private communication with K. Matsuda; see also K. Matsuda, Phys. Rev. D **69**, 113006 (2004) [hep-ph/0401154]
12. For the type-II seesaw mechanism, B. Bajc, G. Senjanović, F. Vissani, Phys. Rev. Lett. **90**, 051802 (2003); H.S. Goh, R.N. Mohapatra, S.P. Ng, Phys. Lett. B **570**, 215 (2003) [hep-ph/0308197]
13. T.E. Clark, T.K. Kuo, N. Nakagawa, Phys. Lett. B **115**, 26 (1982); C.S. Aulakh, R.N. Mohapatra, Phys. Rev. D **28**, 217 (1983)
14. D.G. Lee, Phys. Rev. D **49**, 1417 (1994)
15. C.S. Aulakh, B. Bajc, A. Melfo, G. Senjanović, F. Vissani, Phys. Lett. B **588**, 196 (2004) [hep-ph/0306242]
16. X.G. He, S. Meljanac, Phys. Rev. D **41**, 1620 (1990)
17. T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac, N. Okada, J. Math. Phys. **46**, 033505 (2005) [hep-ph/0405300]
18. G.F. Giudice, A. Masiero, Phys. Lett. B **206**, 480 (1988)
19. N. Sakai, T. Yanagida, Nucl. Phys. B **197**, 533 (1982); S. Weinberg, Phys. Rev. D **26**, 287 (1982)
20. N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi, T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973)
21. S. Dimopoulos, S. Raby, F. Wilczek, Phys. Lett. B **112**, 133 (1982)
22. P. Nath, A.H. Chamseddine, R. Arnowitt, Phys. Rev. D **32**, 2348 (1985)
23. M. Claudson, M.B. Wise, L.J. Hall, Nucl. Phys. B **195**, 297 (1982)
24. S. Chadha, M. Dine, Nucl. Phys. B **229**, 105 (1983)
25. J. Hisano, H. Murayama, T. Yanagida, Nucl. Phys. B **402**, 46 (1993); T. Goto, T. Nihei, Phys. Rev. D **59**, 115009 (1999)
26. S.J. Brodsky, J.R. Ellis, J.S. Hagelin, C.T. Sachrajda, Nucl. Phys. B **238**, 561 (1984)
27. S. Aoki et al. [JLQCD Collaboration], Phys. Rev. D **62**, 014506 (2000)

28. Y. Aoki et al. [RBC Collaboration], Nucl. Phys. Proc. Suppl. **119**, 380 (2003)
29. T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac, N. Okada, JHEP **0409**, 052 (2004) [hep-ph/0406068]
30. V. Lucas, S. Raby, Phys. Rev. D **54**, 2261 (1996)
31. W. Siegel, Phys. Lett. B **84**, 193 (1979)
32. K. Hagiwara et al. [Particle Data Group Collaboration], Phys. Rev. D **66**, 010001 (2002)
33. H.S. Goh, R.N. Mohapatra, S. Nasri, S.P. Ng, Phys. Lett. B **587**, 105 (2004) [hep-ph/0311330]
34. B. Bajc, A. Melfo, G. Senjanović, F. Vissani, Phys. Rev. D **70**, 035007 (2004) [hep-ph/0402122]
35. C.S. Aulakh, A. Girdhar, hep-ph/0204097
36. Z. Maki, M. Nakagawa, S. Sakata, Prog. Theor. Phys. **28**, 870 (1962)
37. J. Gunion, H. Haber, Nucl. Phys. B **272**, 1 (1986); B **278**, 449 (1986)